UNIT-IV

ELECTRIC TRACTION-I

SYSTEM OF ELECTRIC TRACTION:

Basing on supply sources electric traction may be divided into two groups:

a) Self contained locomotives

b) Electric vehicle fed from the distribution network

Self contained locomotives:

In this type, the locomotives itself has a provision for generation of electrical energy required for traction purpose. This group is further is subdivided into :

a)Diesel electric trains& ships

b)Petrol Electric trucks & lorries

c)Battery driven road vehicles

i) DIESEL ELECTRIC TRAINS:

In diesel electric locomotives, the diesel engine is coupled to d.c generator which feeds the electric motors for producing necessary propelling torque . The generator used for this purpose may be self or separately excited. If separately excited is employed, then the excitation is employed , then the excitation is obtained from an auxiliary generator connected in parallel with the battery. The diesel engine employed is usually high speed type with three or four different running speeds with different outputs power. The diesel engine has constant torque characteristic which is undesirable for traction system as more tractive efforts are required by locomotive while moving up gradient. This constant torque characteristic is modified by employing a torque converter which must be included in the transmission between the engine shaft and driving axles.

This system is adopted in areas where coal for stream traction is not available in plenty and water is sometimes scarce.

There are two types of transmission systems which are usually employed in diesel electric trains .They are:

a) MECHANICAL TRANSMISSION:

This system employs a variable gear fixed between engine shaft and driving wheels. This systems of transmission of power is light and effective and is universally employed for light railways vehicle .but for railways work above 300KW output, it is not suitable due to mechanical difficulties with gears and gear changing equipment.

b) ELECTRICAL TRANSIMISSION :

This system employees a diesel engine coupled to d.c generators which supplies current to driving motors assuming the diesel engine is running at constant spped and the generator is giving a consant voltage any increase in tractive effort will reduce the speed of the traction motors owing to their characteristics .The reduction in speed will not be sufficient to compensate for the increased tractive effort and therefore output power required will increased and the engine will be overloaded.

In order that the power delivered by diesel remains constant the product of tractive effort i.e Te*N must remain constant.

Thus the generator voltage must be varied inverserly as the current in order to keep the output power required constant.

ADVANTAGES:

1. Initial cost is low because of the absence of substation ,distribution line feeder etc.
2. It has high acceleration and braking retardation as compared to strem locomotives.
3. The torque exerted is higher than steam engine hence it has more accommodation .
4. There is no loss of power during speed control.
5. Time required for maintenanace and repairs is comparatively less than that of steam locomotives

DISADVANTAGES:

As diesel engine is used along with a d.c generator ,the disadvantages are mainly due to diesel engine i.e,

1. The life of diesel engine is shorter and more over the diesel engine cannot be overloaded even for a short period.

2. For cooling the diesel engine and motor generator set separate cooling system are required .

3. Running and maintenance cost is high.

4. The dead weight on loco will be comparatively more because of motor generator and in order to handle this weight a number of axles are required .

PETROL ELECTRIC TRUCKS AND LORRIES:

This type of traction system is employed for road transport i.e. heavy lorries and buses .Due to electric conversion ,it provides a very fine and continuous control which makes the vehicle capable of moving slowly at an imperceptible speed and creeping up the steepest slope without throttling the engine.

ADVANTAGES :

1. Initial cost is low
2. It is a self contained unit.
3. Speed control and braking system is simple.

DISADVANTAGES :

1. The fuel used for this system has to be imported which affects the country's economy thereby the running and maintenance costs are high.

2. It cannot be overloaded beyond certain limits.
3. The life of propelling equipment used is less as compared to any other drive.

BATTERY DRIVEN:

In this type of traction the propelling unit consists of a d.c motor which is run by secondary batteries.

Due to the unreliability about the source of supply to motor,this method is conveniently employed for frequently operated service such as local delivery of goods in large towns,shunting and traction in industrial works and mines.

ADVANTAGES:

1. Battery driven vehicles is easy to control easy to use.
2. Low maintenance cost & absence of fumes.
3. Less weights.

DISADVANTAGES:

1. Scope of vehicles is limited by small capacity of batteries.
2. Requires frequent charging.
3. Speed range is limited.

ELECTRIC VEHICLE FED FROM DISTRIBUTION NETWORK:

This system of electric traction consist of vehicles which receive electric power from a distributing network fed at suitable points from either a central power station or substation suitably spaced .This system is subdivided into:

i) System operating with d.c such as trolley buses ,tramways and railways.

ii) System operating with a.c such as railways.

SYSTEM OPERATING WITH D.C:

In this system the necessary propelling power is obtained from d.c supply .Some of the examples of such systems are:

TRAMWAYS:

Tramways is a substitute for ordinary buses ,the only difference being they are run on rail tracks .A single overhead conductor having positive polarity is run along the road,the rail track forms the return conductor .The power is fed at suitable points either from central power station or substations. The power is fed at suitable points either from central power stations or substations .The voltage employed is usually 600v.

The tramway are provided with two driving axles on either ends which facilitates in controlling of speed from either ends.Speed of tram can further be increased by weakening the field of motors .Rheostatic and mechanical braking are employed for normal service but mechanical brakes are preferred because they give better retardation due to use of magnetic brakes. The magnetic Brakes usually consist of electromagnets suspended on springs which are attracted to the rail track where they exert sufficient locking force.

Tramway is the most economical means of transportation in large cities.The great disadvantage of tramway is that it requires laying of tracks for its uses which is costly to maintain and constitutes a source of danger to other users.

TROLLEY BUSES:

Serious drawback of tramway is lack of mamoeuvradility i.e movement in congested area and noise as track rails are required to be laid of it and in inner city areas where traffic density is high it will cause difficulty in the movement of traffic .These are overcome by trolley bus drives.

In this type,the use of track rail is avoided. Trolley bus is an electrically operated pneumatic typed vehicle and is fed usually at 600v d.c from two overhead conductors by means of two collectors. A d.c compound motor is usually employeed as speed control is obtained by field weakening (by proving a resistance in shunt field or tappings on series field or diverter with serias field method.

ELECTRIC TRAINS:

Electric trains can be run on a.c & d.c.In earky days d.c at 1,500v &1-φ a.c at 11 to 15kv having frequency 25 c/s or 16 2/3 c/s were used .However d.c was preferred as compared to a.c because in case of a.c an additional equipment is required to convert normal frequency 50 c/s to 25c/s or 16 2/3 c/s.

Later on a.c has proved more advantages in terms of ohmic losses .Moreover in cases of a.c higher voltages can be obtained by use of transformer which reduces the losses and thus increases the spacing between two substations thereby reducing the number of feeding substations .In case of 3-φa.c normal frequency i.e 50c/s is employed for traction thereby by eliminating low frequency difficulty.

COMPARISION BETWEEN D.C TRACTION AND D.C TRACTION:

1.	In D.C traction ,D.C series motor develop more starting and running torque and arc capable of giving high acceleration and retardation.	1.	In A.C trains , starting and running torque developed by a.c motor of same size less and hence acceleration and retardation
2.	Number of speeds obtained by d.c motor is limited except by chopper method	2.	Number of speeds obtained are many by tap changing method
3.	D.C series motor are cheap, lighter &more efficient	3.	A.C motors are somewhat expensive and less efficient
4.	In case of d.c ,the overhead distribution is lighter and less costly as the losses.	4.	In case of a.c ,the overhead distribution is heavy and thus expensive
5.	D.C System produces less interference with communication lines	5.	A.C System produces more interference with communication lines
6.	For a given length of track the number of substations required is more as the voltage above prescribed	6.	For a given length of track the number of substations required is less as the voltage above prescribed
7.	D.C .motors requires less maintenance	7.	A.C .motors requires MORE maintenance

UNIT-7,8

Electric Traction-II,III

INTRODUCTION

The movement of trains and their energy consumption can be most conveniently studied by means of the speed–distance and the speed–time curves. The motion of any vehicle may be at constant speed or it may consist of periodic acceleration and retardation. The speed–time curves have significant importance in traction. If the frictional resistance to the motion is known value, the energy required for motion of the vehicle can be determined from it. Moreover, this curve gives the speed at various time instants after the start of run directly.

TYPES OF SERVICES

There are mainly three types of passenger services, by which the type of traction system has to be selected, namely:

1. Main line service.
2. Urban or city service.
3. Suburban service.

Main line services

In the main line service, the distance between two stops is usually more than 10 km. High balancing speeds should be required. Acceleration and retardation are not so important.

Urban service

In the urban service, the distance between two stops is very less and it is less than 1 km. It requires high average speed for frequent starting and stopping.

Suburban service

In the suburban service, the distance between two stations is between 1 and 8 km. This service requires rapid acceleration and retardation as frequent starting and stopping is required.

SPEED–TIME AND SPEED–DISTANCE CURVES FOR DIFFERENT SERVICES

The curve that shows the instantaneous speed of train in kmph along the ordinate and time in seconds along the abscissa is known as ‘*speed–time*’ curve.

The curve that shows the distance between two stations in km along the ordinate and time in seconds along the abscissa is known as ‘*speed–distance*’ curve.

The area under the speed–time curve gives the distance travelled during, given time internal and slope at any point on the curve toward abscissa gives the acceleration and retardation at the instance, out of the two speed–time curve is more important.

Speed–time curve for main line service

Typical speed–time curve of a train running on main line service is shown in Fig. 10.1. It mainly consists of the following time periods:

1. Constant accelerating period.
2. Acceleration on speed curve.
3. Free-running period.
4. Coasting period.
5. Braking period.

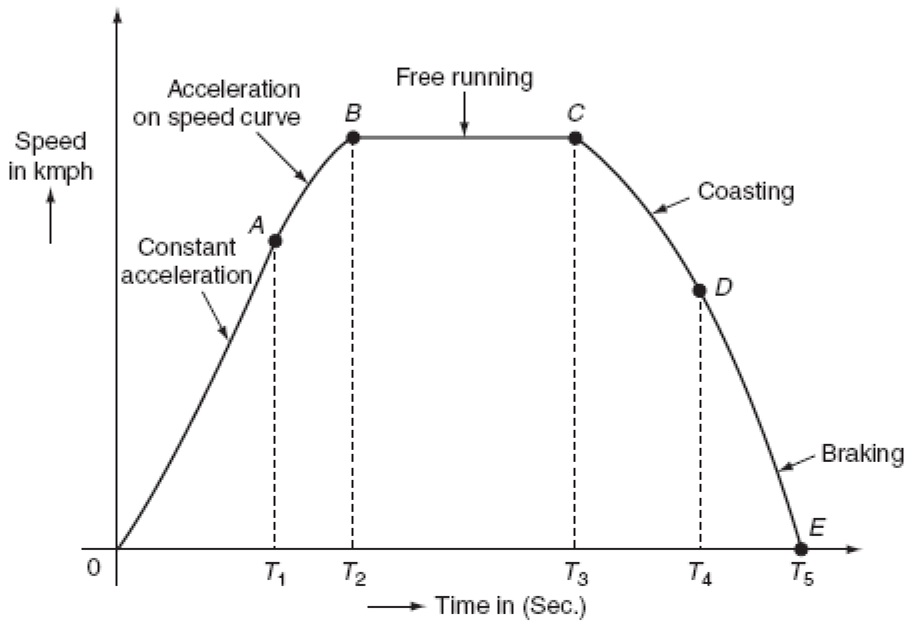

Fig. 10.1 Speed–time curve for mainline service

Constant acceleration

During this period, the traction motor accelerate from rest. The curve 'OA' represents the constant accelerating period. During the instant 0 to T_1, the current is maintained approximately constant and the voltage across the motor is gradually increased by cutting out the starting resistance slowly moving from one notch to the other. Thus, current taken by the motor and the tractive efforts are practically constant and therefore acceleration remains constant during this period. Hence, this period is also called as notch up accelerating period or rehostatic accelerating period. Typical value of acceleration lies between 0.5 and 1 kmph. Acceleration is denoted with the symbol 'α'.

Acceleration on speed-curve

During the running period from T_1 to T_2, the voltage across the motor remains constant and the current starts decreasing, this is because cut out at the instant 'T_1'.

According to the characteristics of motor, its speed increases with the decrease in the current and finally the current taken by the motor remains constant. But, at the same time, even though train accelerates, the acceleration decreases with the increase in speed. Finally, the acceleration reaches to zero for certain speed, at which the tractive effort excreted by the motor is exactly equals to the train resistance. This is also known as decreasing accelerating period. This period is shown by the curve '*AB*'.

Free-running or constant-speed period

The train runs freely during the period T_2 to T_3 at the speed attained by the train at the instant 'T_2'. During this speed, the motor draws constant power from the supply lines. This period is shown by the curve *BC*.

Coasting period

This period is from T_3 to T_4, i.e., from C to D. At the instant 'T_3' power supply to the traction, the motor will be cut off and the speed falls on account of friction, windage resistance, etc. During this period, the train runs due to the momentum attained at that particular instant. The rate of the decrease of the speed during coasting period is known as coasting retardation. Usually, it is denoted with the symbol 'β_c'.

Braking period

Braking period is from T_4 to T_5, i.e., from *D* to *E*. At the end of the coasting period, i.e., at 'T_4' brakes are applied to bring the train to rest. During this period, the speed of the train decreases rapidly and finally reduces to zero.

In main line service, the free-running period will be more, the starting and braking periods are very negligible, since the distance between the stops for the main line service is more than 10 km.

Speed–time curve for suburban service

In suburban service, the distance between two adjacent stops for electric train is lying between 1 and 8 km. In this service, the distance between stops is more than the urban service and smaller than the main line service. The typical speed–time curve for suburban service is shown in Fig. 10.2.

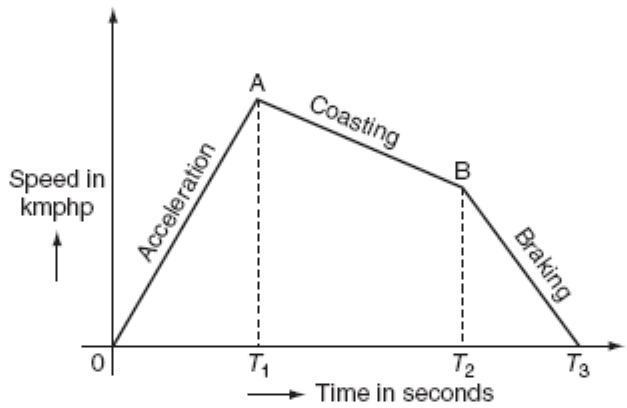

Fig. 10.2 Typical speed–time curve for suburban service

The speed–time curve for urban service consists of three distinct periods. They are:

1. Acceleration.
2. Coasting.
3. Retardation.

For this service, there is no free-running period. The coasting period is comparatively longer since the distance between two stops is more. Braking or retardation period is comparatively small. It requires relatively high values of acceleration and retardation. Typical acceleration and retardation values are lying between 1.5 and 4 kmphp and 3 and 4 kmphp, respectively.

Speed–time curve for urban or city service

The speed–time curve urban or city service is almost similar to suburban service and is shown in Fig. 10.3.

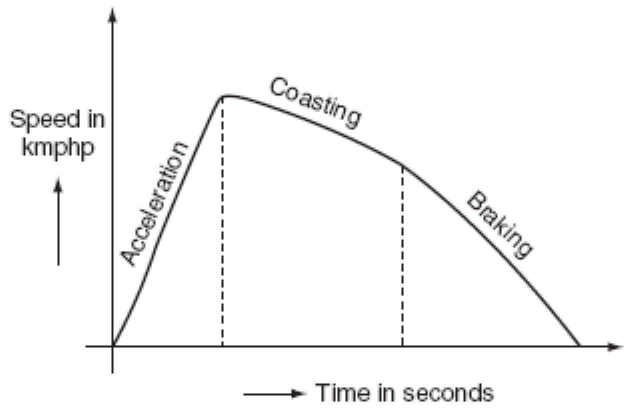

Fig. 10.3 Typical speed–time curve for urban service

In this service also, there is no free-running period. The distance between two stop is less about 1 km. Hence, relatively short coasting and longer braking period is required. The relative values of acceleration and retardation are high to achieve moderately high average between the stops. Here, the small coasting period is included to save the energy consumption. The acceleration for the urban service lies between 1.6 and 4 kmphp. The coasting retardation is about 0.15 kmphp and the braking retardation is lying between 3 and 5 kmphp. Some typical values of various services are shown in Table. 10.1.

Table 10.1 Types of services

	Mainline service	*Suburban service*	*Urban service*
Distance between stops in km	More than 10	1–8	1
Maximum speed in kmph	160	120	120
Acceleration in kmphp	0.5–0.9	1.5–4	1.5–4
Retardation in kmphp	1.5	3–4	3–4
Features	Long free-run period, coasting and acceleration braking periods are small	No free-running period, coasting period is long	No free-running period, coasting period is small

SOME DEFINITIONS

Crest speed

The maximum speed attained by the train during run is known as crest speed. It is denoted with 'V_m'.

Average speed

It is the mean of the speeds attained by the train from start to stop, i.e., it is defined as the ratio of the distance covered by the train between two stops to the total time of rum. It is denoted with 'V_a'.

$$\therefore \text{Average speed} = \frac{\text{distance between stops}}{\text{actual time of run}}$$

$$V_a = \frac{D}{T},$$

where V_a is the average speed of train in kmph, D is the distance between stops in km, and T is the actual time of run in hours.

Schedule speed

The ratio of the distance covered between two stops to the total time of the run including the time for stop is known as schedule speed. It is denoted with the symbol 'V_s'.

$$\therefore \text{Schedule speed} = \frac{\text{distance between stops}}{\text{total time of run} + \text{time for stop}}$$

$$= \frac{\text{distance between stops}}{\text{shedule time}}$$

$$V_s = \frac{D}{T_s},$$

where T_s is the schedule time in hours.

Schedule time

It is defined as the sum of time required for actual run and the time required for stop.

i.e., $T_s = T_{run} + T_{stop}$.

FACTORS AFFECTING THE SCHEDULE SPEED OF A TRAIN

The factors that affect the schedule speed of a train are:

1. Crest speed.
2. The duration of stops.
3. The distance between the stops.
4. Acceleration.
5. Braking retardation.

Crest speed

It is the maximum speed of train, which affects the schedule speed as for fixed acceleration, retardation, and constant distance between the stops. If the crest speed increases, the actual running time of train decreases. For the low crest speed of train it running so, the high crest speed of train will increases its schedule speed.

Duration of stops

If the duration of stops is more, then the running time of train will be less; so that, this leads to the low schedule speed.

Thus, for high schedule speed, its duration of stops must be low.

Distance between the stops

If the distance between the stops is more, then the running time of the train is less; hence, the schedule speed of train will be more.

Acceleration

If the acceleration of train increases, then the running time of the train decreases provided the distance between stops and crest speed is maintained as constant. Thus, the increase in acceleration will increase the schedule speed.

Breaking retardation

High breaking retardation leads to the reduction of running time of train. These will cause high schedule speed provided the distance between the stops is small.

SIMPLIFIED TRAPEZOIDAL AND QUADRILATERAL SPEED TIME CURVES

Simplified speed–time curves gives the relationship between acceleration, retardation average speed, and the distance between the stop, which are needed to estimate the performance of a service at different schedule speeds. So that, the actual speed–time curves for the main line, urban, and suburban services are approximated to some from of the simplified curves. These curves may be of either trapezoidal or quadrilateral shape.

Analysis of trapezoidal speed–time curve

Trapezoidal speed–time curve can be approximated from the actual speed–time curves of different services by assuming that:

- The acceleration and retardation periods of the simplified curve is kept same as to that of the actual curve.
- The running and coasting periods of the actual speed–time curve are replaced by the constant periods.

This known as trapezoidal approximation, a simplified trapezoidal speed–time curve is shown in fig,

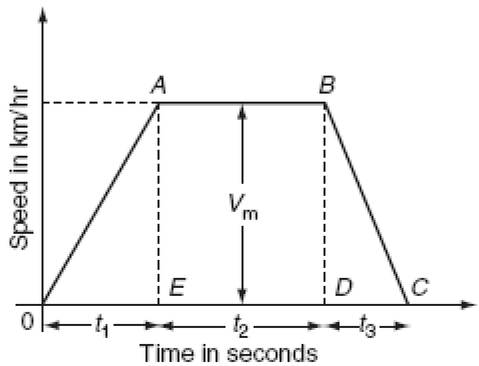

Fig. Trapezoidal speed–time curve

Calculations from the trapezoidal speed–time curve

Let D be the distance between the stops in km, T be the actual running time of train in second, α be the acceleration in km/h/sec, β be the retardation in km/h/sec, V_m be the maximum or the crest speed of train in km/h, and V_a be the average speed of train in km/h. From the Fig. 10.4:

Actual running time of train, $T = t_1 + t_2 + t_3$. **(10.1)**

Time for acceleration, $t_1 = \frac{V_m - 0}{\alpha} = \frac{V_m}{\alpha}$. **(10.2)**

Time for retardation, $t_3 = \frac{V_m - 0}{\beta} = \frac{V_m}{\beta}$. **(10.3)**

$= T - \left[\frac{V_m}{\alpha} + \frac{V_m}{\beta}\right]$. **(10.4)**

Area under the trapezoidal speed–time curve gives the total distance between the two stops (D).

∴ The distance between the stops (D) = area under triangle *OAE* + area of rectangle *ABDE* + area of triangle *DBC*

= The distance travelled during acceleration + distance travelled during free-running period + distance travelled during retardation.

Now:

The distance travelled during acceleration = average speed during accelerating period × time for acceleration

$$= \frac{0+V_m}{2} \times t_1 \text{ km/h} \times \text{sec}$$

$$= \frac{0+V_m}{2} \times \frac{t_1}{3{,}600} \text{ km.}$$

The distance travelled during free-running period = average speed × time of free running

$$= V_m \times t_2 \text{ km/h} \times \text{sec}$$

$$= V_m \times \frac{t_2}{3{,}600} \text{ km.}$$

The distance travelled during retardation period = average speed × time for retardation

$$= \frac{V_m + 0}{2} \times t_3 \text{ km/h} \times \text{sec}$$

$$= \frac{0 + V\text{m}}{2} \times \frac{t_3}{3,600} \text{km.}$$

The distance between the two stops is:

$$D = \frac{V_m}{2} \times \frac{t_1}{3,600} + V_m \times \frac{t_2}{3,600} + \frac{V_m}{2} \times \frac{t_3}{3,600}$$

$$D = \frac{V_m t_1}{7,200} + \frac{V_m}{3,600}\left[T - V_m(t_1 + t_2)\right] + \frac{V_m t_3}{7,200}$$

$$D = \frac{V_m^{\,2}}{7,200\alpha} + \frac{V_m}{3,600}\left[T - V_m\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)\right] + \frac{V_m^{\,2}}{7,200\beta}$$

$$3,600 \times D = \frac{V_m^{\,2}}{2\alpha} + \frac{V_m^{\,2}}{\beta} - V_m^{\,2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + V_m T$$

$$3,600\ D = V_m^{\,2}\left(\frac{1}{2\alpha} - \frac{1}{\alpha}\right) + V_m^{\,2}\left(\frac{1}{2\beta} - \frac{1}{\beta}\right) + V_m T$$

$$3,600\ D = \frac{-V_m^{\,2}}{2\alpha} - \frac{V_m^{\,2}}{2\beta} + V_m T$$

$$\therefore V_m^{\,2}\left[\frac{1}{2\alpha} + \frac{1}{2\beta}\right] - V_m T + 3,600D = 0.$$

$$\text{Let } \frac{1}{2\alpha} + \frac{1}{2\beta} = X = \frac{\alpha + \beta}{2\alpha\beta}$$

$$\therefore V_m^{\,2} X - V_m T + 3,600D = 0. \qquad \textbf{(10.5)}$$

Solving quadratic Equation (10.5), we get:

$$V_m = \frac{T + \sqrt{T^2 - 4 \times X \times 3,600D}}{2 \times X.}$$

$$= \frac{T}{2X} \pm \sqrt{\frac{T^2}{4X^2} - \frac{3,600D}{X}}.$$

By considering positive sign, we will get high values of crest speed, which is practically not possible, so negative sign should be considered:

$$V_m = \frac{T}{2X} - \sqrt{\frac{T^2}{4X^2} - \frac{3,600D}{X}} \quad \textbf{(10.6)}$$

$$\text{Or,} \quad V_m = \frac{\alpha\beta}{\alpha+\beta}T - \sqrt{\left(\frac{\alpha\beta}{\alpha+\beta}\right)^2 T^2 - 7,200\left(\frac{\alpha\beta}{\alpha+\beta}\right)D}.$$

Analysis of quadrilateral speed–time curve

Quadrilateral speed–time curve for urban and suburban services for which the distance between two stops is less. The assumption for simplified quadrilateral speed–time curve is the initial acceleration and coasting retardation periods are extended, and there is no free-running period. Simplified quadrilateral speed–time curve is shown in Fig. 10.5.

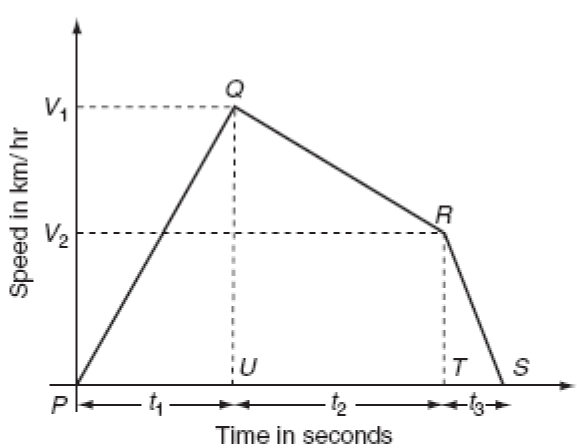

Fig. Quadrilateral speed–time curve

Let V_1 be the speed at the end of accelerating period in km/h, V_2 be the speed at the end of coasting retardation period in km/h, and β_c be the coasting retardation in km/h/sec.

Time for acceleration, $t_1 = \frac{V_1 - 0}{\alpha} = \frac{V_1}{\alpha}$.

Time for coasting period, $t_2 = \frac{V_2 - V_1}{\beta}$.

Time period for braking retardation period, $t_3 = \frac{V_2 - 0}{\beta} = \frac{V_2}{\beta}$.

Total distance travelled during the running period *D*:

= the area of triangle *PQU* + the area of rectangle *UQRS* + the area of triangle *TRS*.

= the distance travelled during acceleration + the distance travelled during coastingretardation + the distance travelled during breaking retardation.

But, the distance travelled during acceleration = average speed × time for acceleration

$$= \frac{0 + V_1}{2} \times t_1 \text{ km/h} \times \text{sec}$$

$$= \frac{V_1}{2} \times \frac{t_1}{3{,}600} \text{ km.}$$

The distance travelled during coasting retardation = $\frac{V_2 + V_1}{2} \times t_2 \text{ km/h} \times \text{sec}$

$$= \frac{V_2 + V_1}{2} \times \frac{t_2}{3,600} \text{km.}$$

The distance travelled during breaking retardation = average speed × time for breaking retardation

$$= \frac{0 + V_2}{2} \times t_3 \text{km/h} \times \text{sec}$$

$$= \frac{V_2}{2} \times \frac{t_3}{3,600} \text{km.}$$

∴ Total distance travelled:

$$D = \frac{V_1}{2} \times \frac{t_1}{3,600} + \frac{(V_1 + V_2)}{2} \frac{(t_2)}{3,600} + \frac{V_2}{2} \times \frac{t_3}{3,600}$$

$$= \frac{V_1 t_1}{7,200} + \frac{(V_1 + V_2) t_2}{7,200} + \frac{V_2 t_3}{7,200}$$

$$= \frac{V_1}{7,200}(t_1 + t_2) + \frac{V_2}{7,200}(t_2 + t_3)$$

$$= \frac{V_1}{7,200}(T - t_3) + \frac{V_2}{7,200}(T - t_1)$$

$$= \frac{(V_1 + V_2)T}{7,200} - \frac{V_1 t_3}{7,200} - \frac{V_2 t_1}{7,200}$$

$$= \frac{(V_1 + V_2)T}{7,200} - \frac{V_1 V_2}{7,200\beta} - \frac{V_1 V_2}{7,200\alpha}$$

$$= \frac{\mathrm{T}}{7,200}(V_1 + V_2) - \frac{V_1 V_2}{7,200}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$7,200D = (V_1 + V_2)\mathrm{T} - V_1 V_2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right). \qquad \textbf{(10.7)}$$

Example 10.1: The distance between two stops is 1.2 km. A schedule speed of 40 kmph is required to cover that distance. The stop is of 18-s duration. The values of the acceleration and retardation are 2 kmphp and 3 kmphp, respectively. Then, determine the maximum speed over the run. Assume a simplified trapezoidal speed–time curve.

Solution:

Acceleration α = 2.0 kmphp.

Retardation β = 3 kmphp.

Schedule speed V_s = 40 kmph.

Distance of run, D = 1.2 km.

$$\text{Schedule time, } T_s = \frac{D \times 3,600}{V_s}$$

$$= \frac{1.2 \times 3,600}{40}$$

$$= 108 \text{ s.}$$

Actual run time, $T = T_s -$ stop duration

= 108 – 18

= 90 s.

$$\text{Maximum speed } V_m = \frac{T}{2X} - \sqrt{\frac{T^2}{4X^2} - \frac{3,600D}{X}},$$

where

$$X = \frac{1}{2\alpha} + \frac{1}{2\beta}$$

$$= \frac{1}{2 \times 2} + \frac{1}{2 \times 3}$$

$$= 0.416.$$

$$\therefore V_{\mathrm{m}} = \frac{90}{2 \times 0.416} - \sqrt{\frac{(90)^2}{4 \times (0.416)^2} - \frac{3,600 \times 1.2}{0.416}}$$

$$= 108.173 - \sqrt{(1,1701.414) - (1,0384.61)}$$

$$= 71.88 \text{ kmph.}$$

Example 10.2: The speed–time curve of train carries of the following parameters:

1. Free running for 12 min.
2. Uniform acceleration of 6.5 kmphp for 20 s.
3. Uniform deceleration of 6.5 kmphp to stop the train.
4. A stop of 7 min.

Then, determine the distance between two stations, the average, and the schedule speeds.

Solution:

Acceleration (α) = 6.5 kmphps.

Acceleration period t_1 = 20 s.

Maximum speed $V_m = \alpha t_1$

$= 6.5 \times 20 = 130$ kmph.

Free-running time (t_2) = 12 × 60

= 720 s.

$$\text{Time for retardation, } (t_3) = \frac{V_{\mathrm{m}}}{\beta}$$

$$= \frac{130}{6.5} = 20 \text{ s.}$$

The distance travelled during the acceleration period:

$$D_1 = \frac{1}{2}\frac{V_m t_1}{3,600}$$
$$= \frac{1}{2} \times \frac{130 \times 20}{3,600}$$
$$= 0.36 \text{ km.}$$

The distance travelled during the free-running period:

$$D_2 = \frac{V_m t_2}{3,600}$$
$$= \frac{130 \times 720}{3,600}$$
$$= 26 \text{ km.}$$

The distance travelled during the braking period $D_3 = \frac{V_m t_3}{7,200}$

$$= \frac{130 \times 20}{7,200}$$
$$= 0.362 \text{ km.}$$

The distance between the two stations:

$D = D_1 + D_2 + D_3$

$= 0.36 + 26 + 0.362$

$= 26.724$ km.

$$\text{Average distance } (V_{avg}) = \frac{D \times 3600}{T}$$

$$= \frac{26.724 \times 3600}{20 + 720 + 20}$$

$$= 126.58 \text{ kmph.}$$

$$\text{Schedule speed } (V_s) = \frac{D \times 3600}{T + \text{stoptime}}$$

$$= \frac{26.724 \times 3,600}{20 + 720 + 20 + 70 \times 60}$$

$$= 81.53 \text{ kmph.}$$

Example 10.3: An electric train is to have the acceleration and braking retardation of 0.6 km/hr/sec and 3 km/hr/sec, respectively. If the ratio of the maximum speed to the average speed is 1.3 and time for stop is 25 s. Then determine the schedule speed for a run of 1.6 km. Assume the simplified trapezoidal speed–time curve.

Solution:

Acceleration α = 0.6 km/hr/s.

Retardation β = 3 km/hr/s.

Distance of run D = 1.6 km.

Let the cultural time of run be 'T' s.

$$\text{Average speed } V_a = \frac{3,600D}{T}$$

$$= \frac{3,600 \times 1.6}{T}$$

$$= \frac{5,760}{T} \text{ kmph.}$$

$$\text{Maximum speed} = 1.3V_a$$

$$= 1.3 \times \frac{5,760}{T}$$

$$= \frac{7,488}{T} \text{ km/hr}$$

$$V_m^2\left[\frac{1}{2\alpha}+\frac{1}{2\beta}\right]-V_{\mathrm{m}}\,T+3,600=D$$

$$V_m^2=\frac{V_{\mathrm{m}}T-3,600D}{\left(\frac{1}{2\alpha}+\frac{1}{2\beta}\right)}$$

$$=\frac{\frac{7,488}{T}\times T-3,600\times 1.6}{\left(\frac{1}{2\times 0.6}+\frac{1}{2\times 3}\right)}$$

$$=\frac{7,488-5,760}{0.833+0.166}$$

$$=1,729.729$$

$$\therefore V_{\mathrm{m}}=41.59 \text{ km/hr.}$$

$$\text{Average speed, } (V_{\mathrm{a}})=\frac{V_{\mathrm{m}}}{1.3}=\frac{41.59}{1.3}$$

$$(V_{\mathrm{a}})=31.9923 \text{ kmph.}$$

$$\text{Actual time of run } T=\frac{3,600D}{V_{\mathrm{a}}}$$

$$=\frac{3,600\times 1.6}{31.9923}$$

$$T=180.0433 \text{ s.}$$

$$\text{Schedule time } T_{\mathrm{s}}=\text{Actual time of run}+\text{time of stop}$$

$$=180.0433+25$$

$$=205.0433 \text{ s.}$$

$$\text{Schedule speed } V_{\mathrm{s}}=\frac{D\times 3,600}{T_{\mathrm{s}}}$$

$$=\frac{1.6\times 3,600}{205.0433}$$

$$=28.0916 \text{ kmph.}$$

Example 10.4: The distance between two stops is 5 km. A train has schedule speed of 50 kmph. The train accelerates at 2.5 kmphps and retards 3.5 kmphps and the duration of stop is 55 s. Determine the crest speed over the run assuming trapezoidal speed–time curve.

Solution:

Acceleration (α) = 2.5 kmphps.

Retardation (β) = 3.5 kmphps.

Distance of run (D) = 5 km.

Schedule speed (V_s) = 50 kmph.

$$\text{Schedule time, } T_s = \frac{D}{V_s} \times 3{,}600$$

$$= \frac{5}{50} \times 3{,}600$$

$$= 360 \text{ s.}$$

$$\text{Actual time of run } T = T_s - \text{Time of stop}$$

$$= 360 - 55$$

$$= 305 \text{ s.}$$

By using the equation:

$$V_m = \frac{T}{2X} - \sqrt{\frac{T^2}{4X2} - \frac{3{,}600D}{X}}$$

$$X = \frac{1}{2\alpha} + \frac{1}{2\beta}$$

$$= \frac{1}{2 \times 2.5} + \frac{1}{2 \times 3.5}$$

$$= 0.2 + 0.1428$$

$$= 0.3428.$$

$$\therefore V_m = \frac{305}{2 \times 0.3428} - \sqrt{\frac{(305)^2}{4 \times (0.3428)^2} - \frac{3600 \times 5}{0.3428}}$$

$$= 444.868 - \sqrt{197{,}905.5898 - 52{,}508.75146}$$

$$= 63.556 \text{ kmph.}$$

Example 10.5: A train is required to run between two stations 1.5 km apart at an average speed of 42 kmph. The run is to be made to a simplified quadrilateral speed–time curve. If the maximum speed is limited to 65 kmph, the acceleration to

2.5, kmphps, and the casting and braking retardation to 0.15 kmphs and 3 kmphs, respectively. Determine the duration of acceleration, costing, and braking periods.

Solution:

Distance between two stations D = 1.5 km.

Average speed V_a = 42 kmph.

Maximum speed V_m = 65 kmph.

Acceleration (α) = 2.5 kmphps.

Coasting retardation β_c = 0.15 kmphps.

Barking retardation β = 3 kmphps.

$$\text{The duration of acceleration } t_1 = \frac{V_m}{\alpha} = \frac{65}{2.5} = 26 \text{ s.}$$

$$\text{The actual time of run, } T = \frac{3,600 \times D}{V_a} = \frac{3,600 \times 1.5}{42} = 128.57 \text{ s.}$$

Before applying brakes; let the speed be V_2.

$$\text{The duration of coasting, } t_2 = \frac{V_m - V_2}{\beta_c} = \frac{65 - V_2}{0.15} \text{ s.}$$

$$\text{The duration of braking } t_3 = \frac{V_2}{\beta} = \frac{V_2}{3} \text{ s.}$$

The actual time of run, $T = t_1 + t_2 + t_3$

$$128.557 = 26 + \frac{65 - V_2}{0.15} + \frac{V_2}{3}$$

$$102.57 = 433.33 - 6.66V_2 + 0.33V_2$$

$$330.76 = 6.33V_2$$

$$V_2 = 52.252 \text{ km/hr.}$$

The duration of coasting, $t_2 = \dfrac{V_m - V_2}{\beta_c}$

$$= \frac{65 - 52.252}{0.15}$$

$$= 84.98 \text{ s.}$$

Duration of braking $(t_3) = \dfrac{V_2}{\beta}$

$$= \frac{52.252}{3}$$

$$= 17.4173 \text{ s.}$$

Example 10.6: A train has schedule speed of 32 kmph over a level track distance between two stations being 2 km. The duration of stop is 25 s. Assuming the braking retardation of 3.2 kmphps and the maximum speed is 20% grater than the average speed. Determine the acceleration required to run the service.

Solution:

Schedule speed V_s = 32 kmph.

Distance D = 2 km.

Duration of stop = 25 s.

Braking retardation = 3.2 kmphps.

$$\text{Schedule time } = \frac{D}{V_s}$$

$$= \frac{2}{32} \times 60 \times 60 = 225 \text{ s.}$$

Actual time of run $T = 225 - 25 = 200$ s.

$$\text{Average speed, } V_a = \frac{3,600 \times D}{T}$$

$$= \frac{3,600 \times 2}{200}$$

$$= 36 \text{ kmph.}$$

$$\text{Maximum speed, } (V_m) = 1.2\ V_a$$

$$= 1.2 \times 36$$

$$V_m = 43.2 \text{ kmph}$$

$$\therefore V_3^2\left(\frac{1}{2\alpha} + \frac{1}{2\beta}\right) - V_m\ T + 3,600$$

$$\frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{V_m T - 3,600 \times D}{V_m^2}$$

$$= \frac{43.2 \times 200 - 3,600 \times 2}{(43.2)^2}$$

$$\frac{1}{2\alpha} + \frac{1}{2\beta} = 0.7716$$

$$\frac{1}{2\alpha} = 0.716 - \frac{1}{2 \times 3.2}$$

$$\frac{1}{2\alpha} = 0.61535$$

$$\alpha = 0.893 \text{ kmphps.}$$

Example 10.7: A suburban electric train has a maximum speed of 75 kmph. The schedule speed including a station stop of 25 s is 48 kmph. If the acceleration is 2 kmphps, the average distance between two stops is 4 km. Determine the value of retardation.

Solution:

Maximum speed V_m = 75 kmph.

The distance of run (D) = 4 km.

Schedule speed (V_s) = 48 kmph.

Acceleration (α) = 2 kmphps.

The duration of stop = 25 s.

$$\text{Schedule time } (T_s) = \frac{D}{V_s}$$

$$= \frac{4}{48} \times 60 \times 60 = 300 \text{ s}.$$

$$\therefore V_m^2\left(\frac{1}{2\alpha} + \frac{1}{2\beta}\right) - V_m T + 3,600 \times D = 0$$

$$\frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{V_m T - 3,600D}{V_m^2}$$

$$\frac{1}{2 \times 2} + \frac{1}{2\beta} = \frac{75 \times 275 - 3,600 \times 4}{(75)^2}$$

$$0.25 + \frac{1}{2\beta} = 1.1066$$

$$\beta = 0.5836 \text{ kmphps.}$$

Example 10.8: An electric train is accelerated at 2 kmphps and is braked at 3 kmphps. The train has an average speed of 50 kmph on a level track of 2,000 min between the two stations. Determine the following:

1. Actual time of run.
2. Maximum speed.
3. The distance travelled before applying brakes
4. Schedule speed.

Assume time for stop as 12 s. And, run according to trapezoidal.

Solution:

Acceleration (α) = 2 kmphps.

Retardation (β) = 3 kmphps.

Average speed (V_a) = 50 kmph.

Distance D = 2,000 min = 2 km.

The duration of stop = 12 s.

(i) Time of run $T = \frac{D}{V_a}$

$$= \frac{2}{50} \times 60 \times 60 = 144 \text{ s.}$$

(ii) Maximum speed, $V_m = \frac{T}{2X} - \sqrt{\frac{T^2}{4X^2} - \frac{3,600D}{X}}$,

where

$$X = \frac{1}{2\alpha} + \frac{1}{2\beta}$$

$$= \frac{1}{2\times 2} + \frac{1}{2\times 3}$$

$$= 0.4166.$$

$$\therefore V_m = \frac{144}{2\times 0.4166} - \sqrt{\frac{(144)^2}{4\times(0.4166)^2} - \frac{3,600\times 2}{0.4166}}$$

$$= 172.8276 - \sqrt{(29,869.397) - (17,282.765)}$$

$$= 60.63744 \text{ kmph.}$$

(iii) $t_3 = \frac{V_m}{\beta} = \frac{60.63744}{3}$

$$= 20.2148 \text{ s.}$$

$$D_3 = \frac{1}{2} V_m t_3$$

$$= \frac{1}{2} \times 60.63744 \times \frac{20.21248}{60\times 60} = 0.173 \text{ km.}$$

The distance travelled before applying brakes

$D_1 + D_2 = D - D_3$

$= 2 - 0.17 = 1.83$ km.

(iv) Schedule speed $V_s = \dfrac{D}{T + T_{stop}}$

$$= \frac{2}{\dfrac{144+12}{60\times 60}} = 46.153 \text{ kmph.}$$

Example 10.9: An electric train has an average speed of 40 kmph on a level track between stops 1,500 m apart. It is accelerated at 2 kmphps and is braked at 3 kmphps. Draw the speed–time curve for the run.

Solution:

Average speed V_a = 40 kmph.

The distance of run (D) = 1,500 m = 1.5 km.

Acceleration (α) = 2 kmphps.

Retroaction (β) = 3 kmphps.

The time of run $T = \dfrac{D}{V_a}$

$$= \frac{1.5}{40} \times 60 \times 60 = 135 \text{ s.}$$

Using the equation (Fig. P.10.1):

$$V_m = \frac{T}{2X} - \sqrt{\frac{T^2}{4X^2} - \frac{3{,}600 D}{X}},$$

where

$$X = \frac{1}{2\alpha} + \frac{1}{2\beta}$$

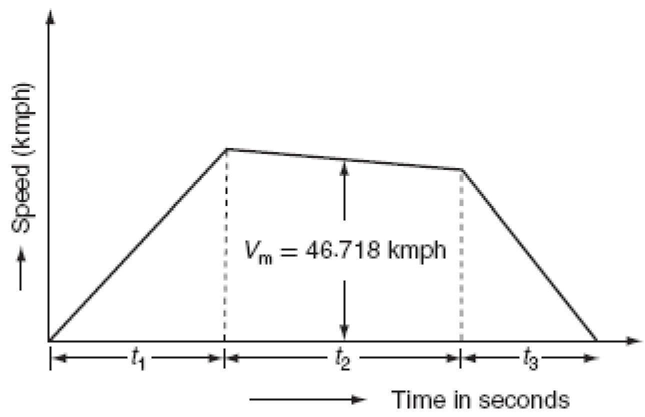

Fig. P.10.1

$$= \frac{1}{2\times 2} + \frac{1}{2\times 3} = 0.416.$$

$$\therefore V_m = \frac{135}{2\times 0.416} - \sqrt{\frac{(135)^2}{4\times(0.416)^2} - \frac{3600\times 1.5}{0.416}}$$

$$= 162.25 - \sqrt{(2,632 - 8.182) - (12,980.769)}$$

$$= 46.718 \text{ kmph.}$$

Acceleration period, $t_1 = \frac{V_m}{\alpha}$

$$= \frac{46.718}{2}$$

$$t_1 = 23.359 \text{ s.}$$

Braking period, $t_3 = \frac{V_m}{\beta}$

$$= \frac{46.718}{3} = 15.572.$$

Free-running period, $t_2 = T - (t_1 + t_3)$

$$= 135 - (23.359 + 15.572)$$

$$= 96.069.$$

Example 10.10: An electric train has quadrilateral speed–time curve as follows:

1. Uniform acceleration from rest at 1.5 kmphps for 25 s.
2. Coasting for 45 s.
3. The duration of braking 20 s.

If the train is moving a uniform up gradient of 1.5%, the reactive resistance is 45 N/ton, the rotational inertia effect is 10% of dead weight, the duration of stop is 15 s, and the overall efficiency of transmission gear and motor is 80%. Find schedule speed.

Solution:

Time for acceleration t_1 = 25 s.

Time for coasting t_2 = 45 s.

Time for braking t_3 = 20 s.

Acceleration (α) = 1.5 kmphps.

Maximum speed Vm = $\alpha\ t_1$

$$= 1.5 \times 25 = 37.5 \text{ kmph.}$$

According to the equation:

$$F_t = 277.8\ W_e(-\beta_c) + 98.1\ WG + Wr$$

$$0 = -\ 277.8 \times 1.1\ W\beta_c + 98.1 \times 1.5 \times W + 45 \times W$$

$$= -305.58\ W\beta_c + 147.15\ W + 45\ W$$

$$30.58\ W\beta_c = 192.15\ W$$

$$\beta_c = \frac{192.15\,W}{305.58\,W}$$

$$\beta_c = 0.628 \text{ kmphps.}$$

The speed at the end of coating period $V_2 = V_m - \beta_c\, t_2$

$$= 37.5 - 0.628 \times 45$$

$$= 9.24 \text{ kmph.}$$

The braking retardation $\beta = \dfrac{V_2}{t_3}$

$$= \frac{9.24}{20} = 0.462 \text{ kmphps.}$$

The distance travelled $D = \dfrac{V_m t_1}{7,200} + \dfrac{(V_m + V_2)t_2}{7,200} + \dfrac{V_2 t_3}{7,200}$

$$= \frac{37.5 \times 25}{7,200} + \frac{(37.5 + 9.24) \times 45}{7,200} + \frac{9.24 \times 20}{7,200}$$

$$= 0.13 + 0.292 + 0.0256$$

$$= 0.4475 \text{ km.}$$

The schedule time $T_s = t_1 + t_2 + t_3 +$ duration of stop

$$= 25 + 45 + 20 + 15$$

$$= 105 \text{ s.}$$

The schedule speed $V_s = \dfrac{3,600 \times D}{T_s}$

$$= \frac{3,600 \times 0.4476}{105}$$

$$V_s = 15.346 \text{ kmph.}$$

TRACTIVE EEFFORT (FT)

It is the effective force acting on the wheel of locomotive, necessary to propel the train is known as '*tractive effort*'. It is denoted with the symbol F_t. The tractive effort is a vector quantity always acting tangential to the wheel of a locomotive. It is measured in newton.

The net effective force or the total tractive effort (F_t) on the wheel of a locomotive or a train to run on the track is equals to the sum of tractive effort:

1. Required for linear and angular acceleration (F_a).
2. To overcome the effect of gravity (F_g).
3. To overcome the frictional resistance to the motion of the train (F_r).

$$\therefore F_t = F_a + F_g + F_r. \qquad \textbf{(10.8)}$$

Mechanics of train movement

The essential driving mechanism of an electric locomotive is shown in Fig. 10.6. The electric locomotive consists of pinion and gear wheel meshed with the traction motor and the wheel of the locomotive. Here, the gear wheel transfers the tractive effort at the edge of the pinion to the driving wheel.

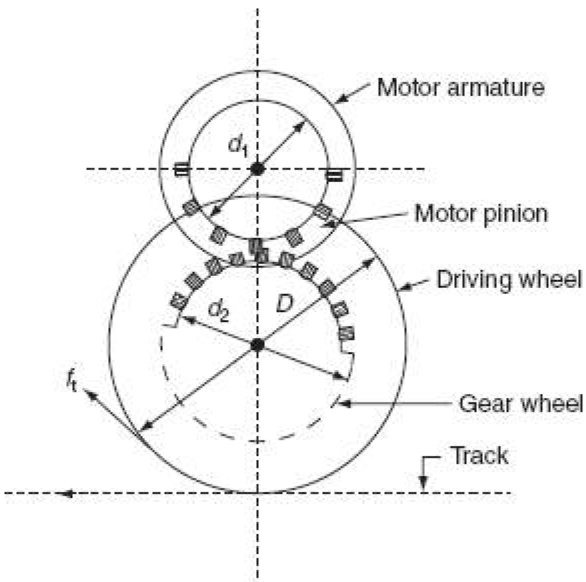

Fig. Driving mechanism of electric locomotives

Let T is the torque exerted by the motor in N-m, F_p is tractive effort at the edge of the pinion in Newton, F_t is the tractive effort at the wheel, D is the diameter of the driving wheel, d_1 and d_2 are the diameter of pinion and gear wheel, respectively, and η is the efficiency of the power transmission for the motor to the driving axle.

Now, the torque developed by the motor $T = F_p \times \frac{d_1}{2}$ N-m.

$$\therefore F_p = \frac{2T}{d_1} \text{ N.} \qquad \textbf{(10.9)}$$

The tractive effort at the edge of the pinion transferred to the wheel of locomotive is:

$$F_t = F_p \times \frac{d_2}{D} \text{ N.} \qquad \textbf{(10.10)}$$

From Equations (10.9) and (10.10) $F_t = \eta \times \frac{2T}{d_1} \times \frac{d_2}{D}$

$$= \eta \cdot T \cdot \frac{2}{D}\left(\frac{d_2}{d_1}\right)$$

$$= \eta T \cdot \frac{2}{D} \cdot r,$$

where 'r'$= \left(\frac{d_2}{d_1}\right)$ is known as gear ratio.

$$\therefore F_t = 2\eta r \frac{T}{D} \text{ N.} \qquad \textbf{(10.11)}$$

10.7.2 Tractive effort required for propulsion of train

From Equation (10.8), the tractive effort required for train propulsion is:

$F_t = F_a + F_g + F_r$,

where F_a is the force required for linear and angular acceleration, F_g is the force required to overcome the gravity, and F_r is the force required to overcome the resistance to the motion.

Force required for linear and angular acceleration (Fa)

According to the fundamental law of acceleration, the force required to accelerate the motion of the body is given by:

Force = Mass × acceleration

$F = ma.$

Let the weight of train be 'W ' tons being accelerated at 'α' kmphps:

$\therefore$ The mass of train $m = 1{,}000$ W kg.

And, the acceleration $= \alpha$ kmphps

$$= \alpha \times \frac{1{,}000}{3{,}600} \text{ m/s}^2$$

$$= 0.2788\alpha \text{ m/s}^2.$$

The tractive effort required for linear acceleration:

$$F_a = 1{,}000 \text{ W kg} \times 0.2778\alpha \text{ m/s}^2$$

$$= 27.88 \text{ W}\alpha \text{ kg} - \text{m/s}^2 \text{ (or) N.} \quad \textbf{(10.12)}$$

Equation (10.12) holds good only if the accelerating body has no rotating parts. Owing to the fact that the train has rotating parts such as motor armature, wheels, axels, and gear system. The weight of the body being accelerated including the rotating parts is known as *effective weight* or *accelerating weight*. It is denoted with 'W_e'. The accelerating weight '(W_e)' is much higher (about 8–15%) than the dead weight (W) of the train. Hence, these parts need to be given angular acceleration at the same time as the whole train is accelerated in linear direction.

$\therefore$ The tractive effort required-for linear and angular acceleration is:

$$F_a = 27.88\ W_e\alpha \text{ N.} \quad \textbf{(10.13)}$$

Tractive effort required to overcome the train resistance (Fr)

When the train is running at uniform speed on a level track, it has to overcome the opposing force due to the surface friction, i.e., the friction at various parts of the rolling stock, the fraction at the track, and also due to the wind resistance. The magnitude of the frictional resistance depends upon the shape, size, and condition of the track and the velocity of the train, etc.

Let '*r*' is the specific train resistance in N/ton of the dead weight and '*W*' is the dead weight in ton.

$\therefore$ The tractive effort required to overcome the train resistance $F_r = Wr$ N. **(10.14)**

Tractive effort required to overcome the effect of gravity (Fg)

When the train is moving on up gradient as shown in Fig. 10.7, the gravity component of the dead weight opposes the motion of the train in upward direction. In order to prevent this opposition, the tractive effort should be acting in upward direction.

∴ The tractive effort required to overcome the effect of gravity:

$$F_g = \pm \text{ mg } \sin\theta \text{ N}$$
$$= \pm 1{,}000 \text{ Wg } \sin\theta \quad [\because m = 1{,}000 \text{ Wkg}]. \qquad \textbf{(10.15)}$$

Now, from the Fig. 10.7:

$$\text{Gradient } = \sin\theta = \frac{BC}{AC} = \frac{\text{Elevation}}{\text{distance along the track}}$$

% Gradient $G = \sin\theta \times 100$. **(10.16)**

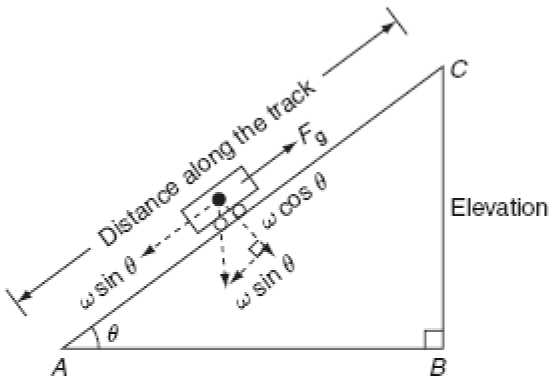

Fig. 10.7 Train moving on up gradient

From Equations (10.15) and (10.16):

$$\therefore F_g = \pm 1,000 \text{ W g} \times \frac{G}{100}$$
$$= \pm\ 10 \times 9.81 \text{ W}G$$
$$= \pm\ 98.1 \text{ W}G \text{ N} \qquad [\text{since } g = 9.81 \text{ m/s}^2]. \quad \textbf{(10.17)}$$

+ve sign for the train is moving on up gradient.

–ve sign for the train is moving on down gradient.

This is due to when the train is moving on up a gradient, the tractive effort showing Equation (10.17)will be required to oppose the force due to gravitational force, but while going down the gradient, the same force will be added to the total tractive effort.

∴ The total tractive effort required for the propulsion of train $F_t = F_a + F_r \pm F_g$:

$$F_t = 277.8\ W_e\alpha + Wr \pm 98.1 \text{ W}G \text{ N}. \qquad \textbf{(10.18)}$$

Power output from the driving axle

Let F_t is the tractive effort in N and v is the speed of train in kmph.

∴ The power output (P) = rate of work done

$$= \text{Tractive effort} \times \frac{\text{distance}}{\text{time}}$$

$$= \text{Tractive effort} \times \text{speed}$$

$$= \frac{F_t \times \nu \times 1{,}000}{3{,}600}\,\text{W}$$

$$= \frac{F_t \times \nu}{3{,}600}\,\text{kW}. \qquad \textbf{(10.19)}$$

If 'v' is in m/s, then $P = F_t \times v$ W.

If 'η' is the efficiency of the gear transmission, then the power output of motors, $P = \frac{F_t \nu}{\eta}\,\text{W:}$:

$$= \frac{F_t \nu}{3{,}600\eta}\,\text{kW}. \qquad \textbf{(10.20)}$$

SPECIFIC ENERGY CONSUMPTION

The energy input to the motors is called the *energy consumption*. This is the energy consumed by various parts of the train for its propulsion. The energy drawn from the distribution system should be equals to the energy consumed by the various parts of the train and the quantity of the energy required for lighting, heating, control, and braking. This quantity of energy consumed by the various parts of train per ton per kilometer is known as specific energy consumption. It is expressed in watt hours per ton per km.

$$\therefore \left.\begin{matrix}\text{Specific energy}\\ \text{consumption}\end{matrix}\right\} = \frac{\text{total energy consumption in W} - \text{h}}{\text{the weight of the train in tons} \times \text{the distance covered by train in km}}$$

10.8.1 Determination of specific energy output from simplified speed–time curve

Energy output is the energy required for the propulsion of a train or vehicle is mainly for accelerating the rest to velocity 'V_m', which is the energy required to overcome the gradient and track resistance to motion.

Energy required for accelerating the train from rest to its crest speed 'Vm'

The energy required for accelerating the train = power × time

$$= \frac{\text{work done}}{\text{time}} \times \text{time}$$

$$= \text{tractive effort} \times \text{velocity} \times \text{time}$$

$$= F_t \times \frac{V_m}{3,600} \times t_1 \text{ N-km/h-sec}$$

$$= F_t \times \frac{1}{2} \times \frac{V_m}{3,200} \times \frac{t_1}{3,600} \text{N-km (or) kW-hr}$$

$$= \frac{1}{2} \times \frac{V_m^{\ 2}}{(3,600)^2\,\alpha} F_t \text{ kw-hr} \left[\because t_1 = \frac{V_m}{\alpha}\right]$$

$$= \frac{1}{2} \times \frac{V_m^{\ 2}}{(3,600)^2\,\alpha}\left[277.8W_e\alpha + 98.1\,WG + Wr\right]\text{kW-hr.}$$

$$[\because F_t = 277.8W_e\,\alpha + 98.1\,WG + Wr].$$

Energy required for overcoming the gradient and tracking resistance to motion

Energy required for overcoming the gradient and tracking resistance:

$$= \text{tractive effort} \times \text{velocity} \times \text{time}$$

$$= F_t' \times \frac{V_m}{3,600} \times \frac{t_2}{3,600} \text{kW-hr}$$

$$= \frac{V_m t_2}{(3,600)^2}\left[Wr + 98.1\,WG\right]\text{kW-hr,}$$

where F_t' is the tractive effort required to overcome the gradient and track resistance, W is the dead weight of train, r is the track resistance, and G is the percentage gradient.

Total energy output = energy required for acceleration + energy required to overcome gradient and to resistance to motion.

$$= \frac{V_m^2}{2(3,600)^2 \alpha}[277.8\, W_e\alpha + 98.1\, WG + Wr] + \frac{V_m t_2}{(3,600)^2}[Wr + 98.1\, WG]\,\text{kW-hr}$$

$$= \frac{V_m^2 (1,000)}{2(3,600)^2 \alpha}[277.8\, W_e\alpha + 98.1\, WG + Wr] + \frac{V_m t_2 \times 1,000}{(3,600)^r}[Wr + 98.1\, WG]\,\text{W-hr}$$

$$= \frac{V_m^2 (1,000)}{2\alpha(3,600)^2}[27.8\, W_e\alpha] + \left[\frac{V_m^2 (1,000)}{2\alpha(3,600)^2} + \frac{V_m t_2 \times 1,000}{(3,600)^2}\right][Wr + 98.1\, WG]\,\text{W-hr}$$

$$= 0.01072\ \text{W}_e V_m^2 + \frac{1,000}{(3,600)}[Wr + 98.1\, WG]\left[\frac{V_m^2}{2\alpha 3,600} + \frac{V_m t_2}{3,600}\right]\text{W-hr}$$

$$= 0.01072\ \text{W}_e V_m^2 + 0.2778[Wr + 98.1\, WG][D_1 + D_2]\,\text{W-hr},$$

where $D_1 = \dfrac{V_m^2}{2\alpha 3,600} = \dfrac{V_m^2}{7,200\alpha}$.

$$D_2 = \frac{V_m t_2}{3,600}.$$

$$\therefore \text{The specific energy output} = \frac{\text{energy output in Whr}}{\text{weight of train in tons} \times \text{distance of running}}$$

$$= \frac{0.001072 V_m^2 W_e + 0.2778[98.1 WG + Wr][D_1 + D_2]}{W \times D}$$

$$= \frac{0.001072 V_m^2}{D}\left[\frac{W_e}{W}\right] + \left[\frac{98.1\,G + r}{D}\right] \times 0.2778 \times D',$$

where $D' = D_1 + D_2$.

For uniform level track $G = 0$:

$$\therefore \text{The specific energy output} = \frac{0.001072 V_m^2}{D}\frac{W_e}{W} + 0.2778 r \times \frac{D'}{D}\ \text{W-hr/ton-km.}$$

$$\therefore \text{The specific energy consumption} = \frac{\text{specific energy output}}{\text{efficiency of motors}}$$

$$= \frac{0.001072 V_m^2}{\eta D}\frac{W_e}{W} + 0.2778\frac{D'}{D}\frac{r}{\eta}\ \text{W-hr/ton-km.} \qquad \textbf{(10.21)}$$

10.8.2 Factors affecting the specific energy consumption

Factors that affect the specific energy consumption are given as follows.

Distance between stations

From equation specific energy consumption is inversely proportional to the distance between stations. Greater the distance between stops is, the lesser will be the specific energy consumption. The typical values of the specific energy consumption is less for the main line service of 20–30 W-hr/ton-km and high for the urban and suburban services of 50–60 W-hr/ton-km.

Acceleration and retardation

For a given schedule speed, the specific energy consumption will accordingly be less for more acceleration and retardation.

Maximum speed

For a given distance between the stops, the specific energy consumption increases with the increase in the speed of train.

Gradient and train resistance

From the specific energy consumption, it is clear that both gradient and train resistance are proportional to the specific energy consumption. Normally, the coefficient of adhesion will be affected by the running of train, parentage gradient, condition of track, etc. for the wet and greasy track conditions. The value of the coefficient of adhesion is much higher compared to dry and sandy conditions.

IMPORTANT DEFINITIONS

1 Dead weight

It is the total weight of train to be propelled by the locomotive. It is denoted by 'W'.

2 Accelerating weight

It is the effective weight of train that has angular acceleration due to the rotational inertia including the dead weight of the train. It is denoted by 'W_e'.

This effective train is also known as accelerating weight. The effective weight of the train will be more than the dead weight. Normally, it is taken as 5–10% of more than the dead weight.

3 Adhesive weight

The total weight to be carried out on the drive in wheels of a locomotive is known as adhesive weight.

4 Coefficient of adhesion

It is defined as the ratio of the tractive effort required to propel the wheel of a locomotive to its adhesive weight.

$$F_t \propto W$$

$$= \mu W,$$

where F_t is the tractive effort and W is the adhesive weight.

$$\therefore \mu = \frac{F_t}{W}. \qquad (10.22)$$

Example 10.11: A 250-ton motor coach having four motors each developing 6,000 N-m torque during acceleration, starts from rest. If the gradient is 40 in 1,000, gear ration is 4, gear transmission efficiency is 87%, wheel radius is 40 cm, train resistance is 50 N/ton, the addition of rotational inertia is 12%. Calculate the time taken to attain a speed of 50 kmph. If the line voltage is 3,000-V DC and the efficiency of motors is 85%. Find the current during notching period.

Solution:

The weight of train W = 250 ton.

$$\text{Parentage gradient } G = \frac{40}{1,000} \times 100 = 4\%.$$

Gear ratio r = 4.

Wheel diameter D = 2 × 40 = 80 cm.

Or, D = 0.8 m.

Train resistance r = 50 N/ton.

Rotational inertia = 12%.

Accelerating weight of the train W_e = 1.10 × 250 = 275 ton.

Total torque developed T = 4 × 6,000 = 24,000 Nm.

$$\text{Tractive effort } F_t = \frac{\eta T 2r}{D}$$

$$= \frac{0.87 \times 24,000 \times 2 \times 4}{0.8} = 208,800\,\text{N}.$$

But,

$F_t = 277.8\ W_e\alpha + 98.1\ WG + Wr$

208,800 = 277.8 × 275 α + 98.1 × 250 × 4 + 250 × 50

$\therefore$ α = 1.285 kmphps.

The time taken for the train to attain the speed of 50 kmph:

$$t = \frac{V_m}{\alpha}$$
$$= \frac{50}{1.285} = 38.89\,\text{s}.$$

Power output from the driving axles:

$$= \frac{F_t \times V_m}{3,600} = \frac{208,800 \times 50}{3,600}$$
$$= 2,900\ \text{kW}.$$

$$\text{Power input} = \frac{\text{power output}}{\eta_m}$$
$$= \frac{2,900}{0.85} = 3,411.76\,\text{kW}.$$

$$\text{Total current drawn} = \frac{\text{power input}}{V}$$
$$= \frac{3,411.76 \times 10^3}{3,000} = 1,137.25\,\text{A}.$$

$$\text{Current drawn by the each motor} = \frac{1,137.25}{4} = 284.31\,\text{A}.$$

Example 10.12: An electric train of weight 250 ton has eight motors geared to driving wheels, each is 85 cm diameter. The tractive resistance is of 50/ton. The effect of rotational inertia is 8% of the train weight, the gear ratio is 4–1, and the gearing efficiency is 85% determine. The torque developed by each motor to accelerate the train to a speed of 50 kmph in 30 s up a gradient of 1 in 200.

Solution:

The weight of train W = 250 ton.

The diameter of driving wheel D =0.85 m.

Tractive resistance, r = 50N/ton.

Gear ratio r = 4.

Gearing efficiency η = 0.85.

Accelerating weight of the train:

W_e = 1.10 × W

= 1.10 × 250 =275 ton.

Maximum speed V_m = 50 kmph.

$$\text{Acceleration } \alpha = \frac{V_m}{t_1} = \frac{50}{30} = 1.66 \text{ kmpmph.}$$

Tractive effort F_t = 277.8 $W_e \alpha$ + 98.1 WG + Wr

= 126,815.7+12,262.5+12,500

= 151,578.2 N.

$$\text{Total torque developed } T = \frac{F_t \times D}{\eta \times 2\gamma}$$

$$= \frac{151,578.2 \times 0.85}{0.85 \times 2 \times 4}$$

$$= 18.947.25 \text{ N-m.}$$

$$\text{Torque developed by each motor} = \frac{18,947.25}{8}$$

$$= 2,368.409 \text{ N-m.}$$

Example 10.13: A tram car is equipped with two motors that are operating in parallel, the resistance in parallel. The resistance of each motor is 0.5 Ω. Calculate the current drawn from the supply mains at 450 V when the car is running at a steady-state speed of 45 kmph and each motor is developing a tractive effort of 1,600 N. The friction, windage, and other losses may be assumed as 3,000 W per motor.

Solution:

The resistance of each motor = 0.5 Ω.

Voltage across each motor V = 450 V.

Tractive effort F_t = 1,600 N.

Maximum speed V_m = 45 kmph.

Losses per motor = 3,000 W.

$$\text{The power output of each motor} = \frac{F_t \times V_m}{3,600}$$
$$= \frac{1,600 \times 45 \times 10^3}{3,600}$$
$$= 20,000 \text{ W.}$$

Copper losses = $I^2 R_m = I^2 \times 0.5$

Motor input = motor output + constant loss + copper losses

$450 \times I = 20,000 + 3,000 + 0.5I^2$

$0.5\,I^2 - 450I + 23,000 = 0.$

After solving, we get I = 54.39 A.

Total current drawn from supply mains = 2 × 54.39

= 108.78 A.

Example 10.14: A locomotive exerts a tractive effort of 35,000 N in halting a train at 50 kmph on the level track. If the motor is to haul the same train on a gradient of 1 in 50 and the tractive effort required is 55,000 N, determine the power delivered by the locomotive if it is driven by (i) DC series motors and (ii) induction motors.

Solution:

Tractive effort F_t= 35,000 N.

Maximum speed V_m = 50 kmph.

$$\text{Power output} = \frac{F_t \times V_m}{3,600}$$
$$= \frac{35,000 \times 50 \times 10^3}{3,600}$$
$$= 486,111.11 \text{ W}$$
$$= 486.11 \text{ kW.}$$

The power delivered by the locomotive on up gradient track with the DC series motors:

$$= 486.11\sqrt{\frac{55,000}{35,000}}$$
$$= 609.37 \text{ kW.}$$

Since the power output $\propto \sqrt{T} \propto \sqrt{F_t}$, the power delivered by the locomotive on up gradient with the induction motors is:

$$= 486.11 \times \frac{55,000}{35,000}$$
$$= 763.8875 \text{ W} \quad (\because \text{power output} \propto T \propto F_t).$$

Example 10.15: A train weighting 450 ton has speed reduced by the regenerative braking from 50 to 30 kmph over a distance of 2 km on down gradient of 1.5%. Calculate the electrical energy and the overage power returned to the line tractive resistance is 50 N/ton. And, allow the rotational inertia of 10% and the efficiency conversion 80%.

Solution:

The accelerating weight of the train W_e = 1.1 W

= 1.1 × 450 = 495 ton.

The distance travelled D = 2 km.

Gradient G = 1.5%

Track resistance r = 50 N/ton.

Efficiency η = 0.8.

The energy available due to the reduction in the speed is:

$$= 0.01072\, W_e V_1^2 - V_2^2$$
$$= 0.1072 \times 495\ (50^2 - 30^2)$$
$$= 8{,}490.24 \text{ W-hr}$$
$$= 8.49 \text{ kW-hr.}$$

The tractive effort required while going down the gradient:

F_t = Wr – 98.1 WG

= 450 × 50 – 98.1 × 450 × 1.5

= -43,717.5 N.

The energy available while moving down the gradient a distance of 2 km is:

$$\frac{F_t \times D \times 1{,}000}{1{,}000 \times 3{,}600} \text{ kW-hr}$$
$$= \frac{43{,}717.5 \times 2 \times 1{,}000}{1{,}000 \times 3{,}600}$$
$$= 24.2875 \text{ kW-hr.}$$

The total energy available = 8.49 + 24.2875

= 32.7775 kW-hr.

$$\text{The average speed } = \frac{50+30}{2}$$
$$= 40 \text{ kmph.}$$

$$\text{The time taken to cover } 2\,\text{km} = \frac{2}{40} = \frac{1}{20}\,\text{h.}$$

$$\text{The average power} = \frac{\text{Energy returned to the line}}{\text{time}} = \frac{26.222}{1/20} = 524.44 \text{ kW.}$$

Example 10.16: A train weighing 450 ton is going down a gradient of 20 in 1,000, it is desired to maintain train speed at 50 kmph by regenerative braking. Calculate the power fed into the line and allow rotational inertia of 12% and the efficiency of conversion is 80%. Traction resistance is 50 N/ton.

Solution:

The dead weight of train W = 450 ton.

The maximum speed V_m = 50 kmph.

$$\text{Gradient G} = \frac{20 \times 100}{1,000} = 2\%.$$

Tractive resistance r = 50 N/ton.

Rotational inertia = 12%.

The efficiency of conversion = 0.8

The tractive effort required while going down the gradient:

Tractive resistance r = 50 N/ton.

Rotational inertia = 12%.

The efficiency of conversion = 0.8

The tractive effort required while going down the gradient:

= Wr – 98.1 WG

$= 450 \times 50 - 98.1 \times 450 \times 2$

$= -65{,}790$ N.

$$\text{The power available P} = \frac{F_t \times V_m}{3{,}600}$$

$$= \frac{65{,}790 \times 50}{3{,}600}$$

$$= 913.75 \text{ kW.}$$

The power fed into the line = power available × efficiency of conversion

$= 913.75 \times 0.8$

$= 731$ kW.

Example 10.17: The speed–time curve of an electric train on a uniform raising gradient of 10 in 1,000 comprise of:

1. Uniform acceleration from rest at 2.2 kmphps for 30 s.
2. Wasting with power off for 30 s.
3. Braking at 3.2 kmphps to standstill the weight of the train is 200 ton. The tractive resistance of level track being 4 kg/ton and the allowance for rotary inertia 10%. Calculate the maximum power developed by traction motors and the total distance travelled by the train. Assume the transmission efficiency as 85%.

Solution:

$$\text{Gradient} = \frac{10}{1{,}000} \times 100 = 1\%.$$

Acceleration (α) = 2.2 kmphps.

Braking (β) = 3.2 kmph.

The dead weight of train W = 200 ton.

Track resistance r = 4 kg/ton = 4 × 9.81 = 39.24 N/ton.

Maximum velocity Vm = αt_1 = 2.2 × 30 = 66 kmph.

Tractive effort required:

$F_t = 277.8\ W_e\alpha + 98.1\ WG + Wr$

$= 277.8 \times 8 \times 1.1 \times 200 \times 2.2 + 98.1 \times 200 \times 1 + 200 \times 39.24$

$= 161{,}923.2$ N.

$$\text{The maximum power output} = \frac{F_t V_m}{3{,}600} = \frac{161{,}923.2 \times 66}{3{,}600} = 2{,}968.592 \text{ kW.}$$

The maximum power developed by the traction motor = $\frac{2{,}968.592}{0.85}$ = 3492.46kW.

Let, the coasting retardation be βc:

$F_t = 277.8\ W_e(-\beta_c) + 98.1\ WG + Wr$

$0 = -277.8 \times (1.1 \times 200) \times \beta_c + 98.1 \times 200 \times 1 + 200 \times 39.24$

$\beta_c = 0.449$ kmphps

$V_2 = V_m - \beta_c V_2$

$= 66 - 0.449 \times 65$

$= 36.815$ kmph.

$$\text{Braking period } t_3 = \frac{V_2}{\beta} = \frac{36.815}{3.2} = 11.504 \text{ s.}$$

The total distance travelled by the train:

$$D = \frac{V_m t_1}{7,200} + \frac{(V_1 + V_2)t_2}{7,200} + \frac{V_2 t_3}{7,200}$$

$$= \frac{66 \times 30}{7,200} + \frac{(66 + 36.815) \times 65}{7,200} + \frac{36.815 \times 11.504}{7,200}$$

$$= 0.275 + 0.928 + 0.0588$$

$$= 1.26 \text{ km.}$$

Example 10.18: A 2,300-ton train proceeds down a gradient of 1 in 100 for 5 min, during which period, its speed gets reduced from 40 to 20 kmph by the application of the regenerative braking. Find the energy returned to the lines if the tractive resistance is 5 kg/ton, the rotational inertia 10%, and the overall efficiency of the motors during regeneration is 80%.

Solution:

The dead weight of the train W = 2,300 ton.

The accelerating weight of the train W_e = 1.1 × 2,300 s

= 2,530 ton.

$$\text{Gradient } = \frac{1}{100} \times 100 = 1\%.$$

Tractive resistance r = 5×9.81= 49.05 N/ton.

Regenerative period t = 5 × 60

= 300 s.

Overall efficiency η = 0.8.

The energy available due to the reduction in speed:

= 0.01072 $W_e (V_1^2 - V_2^2)$

= 0.01072 × 2,530 × $(40^2 - 20^2)$

$= 32{,}545.92$

$= 32.54$ kW-hr.

The tractive effort required while going down the gradient:

$= Wr - 98.1\ WG$

$= 2{,}300 \times 49.05 – 98.1 \times 2{,}300 \times 1$

$= –112{,}815.$

The distance moved during regeneration:

$$= \frac{V_1 + V_2}{2} \times \frac{1{,}000}{3{,}600} \times t$$

$$= \frac{40 + 20}{2} \times \frac{1{,}000}{3{,}600} \times 300$$

$$= 2{,}500 \text{ m.}$$

The energy available on the account of moving down the gradient over a distance of 2,500 m:

$$= \frac{112{,}815 \times 2{,}500}{2{,}600 \times 1{,}000}$$

$$= 78.34 \text{ kW-hr.}$$

The total energy available = 32.54 + 78.34

= 88.707 kW-hr.

The energy returned to the line = 0.8×11.08

= 88.707 kW-hr.

Example 10.19: An electric train has an average speed of 50 kmph on a level track betweenstops 1,500 m a part. It is accelerated at 2 kmphs and is braked at 3 kmphs. Estimate the energy consumption at the axle of the train per ton-km. Take the reactive resistance constant at 50 N/ton and allow 10% for rotational inertia.

Solution:

Acceleration (α) = 2 kmphs.

Retardation (β) = 3 kmphs.

The distance of run (D) = 1.5 km.

Average speed V_a = 50 kmph.

$$\text{The time of run } T = \frac{D}{V_a} \times 3,600$$
$$= \frac{1.5}{50} \times 3,600$$
$$= 108 \times 10^3 \text{ s.}$$

Using the equation:

$$V_m = \frac{T}{2X} - \sqrt{\frac{T^2}{4X^2} - \frac{3,600}{X}}$$

$$X = \frac{1}{2\alpha} + \frac{1}{2\beta}$$

$$= \frac{1}{2\times 2} + \frac{1}{2\times 3} = 0.416.$$

$$V_m = \frac{108}{2\times 0.416} - \sqrt{\frac{(108)^2}{4\times(0.416)} - \frac{3600\times 3.5}{0.416}}$$

$$= 129.807 - \sqrt{16850.036 - 1298.769}$$

$$= 67.603 \text{ kmph.}$$

Accelerating period, $t_1 = \frac{V_m}{\alpha}$

$$= \frac{67.603}{2}$$

$$= 33.8015 \text{ s.}$$

Braking period, $t_3 = \frac{V_m}{\beta}$

$$= \frac{67.603}{3}$$

$$= 22.534 \text{ s.}$$

The distance travelled during braking:

$$= 1/2 \times V_m \times \frac{t_3}{3,600}$$

$$= \frac{1}{2} \times 67.603 \times \frac{22.534}{3,600}$$

$$= 0.2115 \text{ km.}$$

$$D_1 = D - 0.2115$$

$$= 1.5 - 0.2115$$

$$= 1.288 \text{ km.}$$

Tractive resistance $r = 50$ N/ton

$$\frac{W_e}{W} = 1.1.$$

The energy consumption at the axle of the train per ton-km:

$$= \frac{0.01072 V_m^2}{D} \times \frac{W_e}{W} + 0.2778 r \frac{D_1}{D}$$

$$= \frac{0.01072 \times (67.603)^2}{1.5} \times 1.1 + 2,778 \times 50 \times \frac{1.288}{1.5}$$

$$= 35.927 + 11.926$$

$$= 47.853 \text{ W-hr.}$$

Example 10.20: An electric train has quadrilateral speed–time curve as follows.

1. The uniform acceleration for rest at 2.2 kmphs for 30 s.
2. Coasting for 45 s.
3. The braking period of 20 s.

The train is moving in a uniform up gradient of 1%, the tractive resistance is 50 N/ton, the rotational inertia effect 10% of the dead weight the duration of the station stop 20 s and overall efficiency of transmission gear and motor as 80%. Determine the value of is schedule speed and specific energy consumption of run.

Solution:

Time of acceleration t_1 = 30 s.

Time of coasting t_2 = 45 s.

Time of braking t_3 = 20 s.

Acceleration (α) = 2.2 kmphps.

Maximum speed $V_m = \alpha\, t_1 = 2.2 \times 30 = 66$ kmph.

Gradient G = 1%.

Let the coasting retardation be β_c:

$$F_t = 277.8\, W_e(-\beta_c) + 98.1\, WG + Wr.$$

$0 = 277.8 \times 1.1\ W\beta_c + 98.1 \times W \times 1 + 50\ W$

$= -305.58\ W\beta_c + 98.1\ W + 50\ W.$

$\beta_c = 0.4846$ kmphps.

$V_2 = V_m - \beta_c t_2$

$= 66 - 0.4846 \times 45$

$= 44.193$ kmph.

Braking retardation, $\beta = \dfrac{V_2}{t_3} = \dfrac{44.193}{20}$

$= 2.207$ kmphps.

The distance travelled $D = \dfrac{V_m t_1}{7,200} + \dfrac{(V_m + V_2)t_2}{7,200} + \dfrac{V_2 t_3}{7,200}$

$= \dfrac{66 \times 30}{7,200} + \dfrac{(66 + 44.193)}{7,200} \times 45 + \dfrac{44.193 \times 20}{7,200}$

$= 0.275 + 0.688 + 0.122$

$= 1.085$ km.

Schedule time, $T_s = t_1 + t_2 + t_3 +$ stop duration

$= 30 + 45 + 20 + 20$

$= 115$ s.

Schedule speed, $V_s = \dfrac{3,600 \times D}{T_s}$

$= \dfrac{3,600 \times 1.085}{115}$

$= 33.965$ kmph.

When power is on, the distance travelled is:

D_1 = distance travelled during acceleration period

$$= \frac{V_m t_1}{7,200}$$

$$= \frac{66 \times 30}{7,200} = 0.275 \text{ km.}$$

The specific energy output:

$$= \frac{0.01072 V_m^{\ 2}}{D} \times \frac{W_e}{W} + 0.2778(98.1G + r)\frac{D_1}{D}$$

$$= \frac{0.01072 \times 66^2}{1.085} \times 1.1 + 0.2778(98.1 \times 1 + 50) \times \frac{0.275}{1.085}$$

$$= 47.341 + 10.427$$

$$= 57.768 \text{ W-hr/ton-km.}$$

$$\text{The specific energy consumption } = \frac{57.768}{0.8}$$

$$= 72.21 \text{ W-hr/ton-km.}$$

Example 10.21: A train weighing 200-ton accelerates uniformly from rest to a speed of 40 kmph up a gradient of 1 in 100, the time taken being 30 s. The power is then cut off and train coasts down a uniform gradient of 1 in 1,000 for period of 40 s. When brakes are applied for period of 20 s so as to bring the train uniformly to rest on this gradient determine:

1. The maximum power output from the driving axles.
2. The energy taken from the conductor rails in kW-hr assuming an efficiency of 70% assume tractive resistance to be 45 N/ton at all speeds and allow 10% for rotational inertia.

Solution:

$$\text{Accelerating weight, } W_e = 1.1 \times 200$$

$$= 220 \text{ ton.}$$

$$\text{Tractive resistance, } r = 45 \text{ N/ton}$$

$$\text{Gradient } = \frac{1}{100} \times 100$$

$$= 1\%.$$

Maximum speed V_m = 40 kmph.

Accelerating period t_1 = 30 s.

$$\text{Acceleration } \alpha = \frac{V_m}{t_1} = \frac{40}{30} = 1.33 \text{ kmphps.}$$

Tractive effort required:

$F_t = 27.88\ W_e\ \alpha + 98.1\ WG + Wr$

= 277.8 × 220 × 1.33 + 98.1 × 200 × 1 + 200 × 45

= 109,904.28 N.

1. The maximum power output from driving axle:

$$= \frac{F_t \times V_m}{3,600} = \frac{109,904 \times 40}{3,600} = 1,221.155 \text{ kW.}$$

Total energy required for the run:

$$= \frac{1}{2} \times \frac{F_t V_m}{3,600} \times \frac{t_1}{3,600} = \frac{1}{2} \times \frac{109,904 \times 40}{3,600} \times \frac{30}{3,600} = 5.088 \text{ kW-hr.}$$

2. $$\text{The energy taken for conductor rails } = \frac{5.088}{0.7} = 7.268 \text{ kW-hr.}$$

Example 10.22: Calculate the energy consumption if a maximum speed of 12 m/sec and for a given run of 1,500 m, an acceleration of 0.36 m/s^2 desired. The tractive resistance during acceleration is 0.052 N/kg and during the coasting is 6.12

N/1,000 kg. Allow a 10% of rotational inertia, the efficiency of the equipment during the acceleration period is 60%. Assume quadrilateral speed–time curve.

Solution:

Accelerating weight of the train W_e = 1.1 W.

Maximum speed V_m = 12 m/s.

The distance of run D = 1,500 m.

Acceleration α = 0.36 m/s2.

$$\text{Accelerating period } t_1 = \frac{V_m}{\alpha} = \frac{12}{0.36} = 33.33 \text{ s.}$$

The tractive resistance during acceleration r = 0.52 N/kg.

The tractive effort required during acceleration $F_t = W_e\alpha + Wr$

= 1.1 W × 0.36 + W × 0.052

= 0.448 W N.

The total energy required for the run = average power during acceleration × accelerating period

$$= \frac{1}{2} F_t V_m \times t_1$$

$$= \frac{1}{2} \times 0.448 \times 12 \times 33.3$$

$$= 89.51 \text{ J}$$

$$= \frac{89.51}{3,600}$$

$$= 0.024 \text{ W-hr.}$$

$$\text{Specific energy output} = \frac{\text{energy required for the run}}{W \times D}$$

$$= \frac{0.024\,W}{W \times 1,500}$$

$$= 1.712 \times 10^{-5} \text{ W-hr/kg-m.}$$

$$\text{Specific energy consumption} = \frac{\text{specific energy output}}{\eta}$$

$$= \frac{1.713 \times 10^{-5}}{0.6}$$

$$= 2.85 \times 10^{-5} \text{ W-hr/kg-m.}$$

Example 10.23: A 100-ton weight train has a rotational inertia of 10%. This train has to be run between two stations that are 3 km a part and has an average speed of 50 km/hr. The acceleration and the retardation during braking are 2 kmphps and 3 kmphps, respectively. The percentage gradient between these two stations is 1% and the train is to move up the incline the track resistance is 50 N/ton, then determine:

1. Maximum power at the driving axle.
2. Total energy consumption.
3. Specific energy consumption.

The combined efficiency of the alembic train is 70%. Assume simplified trapezoidal speed–time curve.

Solution:

The dead weight of the train, W = 100 ton.

The accelerating weight of the train, $W_e = 1.1 \times W = 1.1 \times 100 = 110$ ton.

The distance of run (D) = 3 km.

Average speed V_a = 50 kmph.

Acceleration (α) = 2 kmphps.

Retardation (β) = 3 kmphps.

Gradient (G) = 1%.

Tractive resistance r = 50 N/ton.

$$\text{Duration of run} = \frac{3,600 \times D}{V_a}$$

$$= \frac{3,600 \times 3}{50} = 216\,\text{km},$$

$$\text{where } X = \frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{1}{2 \times 2} + \frac{1}{2 \times 3} = 0.416.$$

Using the equation, the maximum speed:

$$V_m = \frac{T}{2X} - \sqrt{\frac{T^2}{4X^2} - \frac{3,600D}{X}}$$

$$= \frac{216}{2 \times 0.416} - \sqrt{\frac{(216)^2}{4 \times (0.416)^2} - \frac{3,600 \times 3}{0.416}}$$

$$= 259.615 - \sqrt{(67,400.147) - (25,961.538)}$$

$$= 56.05 \text{ kmph.}$$

Accelerating period, $t_1 = \frac{V_m}{\alpha}$

$$= \frac{56.05}{2}$$

$$= 28.025 \text{ s.}$$

Braking period $t_3 = \frac{V_m}{\beta}$

$$= \frac{56.05}{3}$$

$$= 18.683 \text{ s.}$$

Free-running period $t_2 = 216 - (28.025 + 18.683)$

$= 169.292$ s.

Tractive effort required $F_t = 278\ W_e\ \alpha + 98.1\ WG + Wr$

$= 277.8 \times 110 \times 2 + 98.1 \times 100 \times 1 + 100 \times 50$

$= 75,926$ N.

(i) Maximum power at the driving axle:

$$= \frac{F_t \times V_m}{3,600}$$

$$= \frac{75,926 \times 56.05}{3,600}$$

$$= 1,182.125 \text{ kW.}$$

Tractive effort required during free running is F_t^1:

$$F_t^1 = 98.1\,WG + Wr$$
$$= 98.1 \times 100 \times 1 + 100 \times 50$$
$$= 14{,}810 \text{ N.}$$

$$\text{Total energy output} = \frac{1}{2} F_t \times \frac{V_m}{3,600} \times \frac{t_1}{3,600} + \frac{F_t \times V_m}{3,600} \times \frac{t_2}{3,600}$$
$$= \frac{1}{2} \times 75,926 \times \frac{56.05}{3,600} \times \frac{28.025}{3,600} + \frac{14,810 \times 56.05}{3,600} \times \frac{169.292}{3,600}$$
$$= 4.6 + 10.843$$
$$= 15.44 \text{ kW-hr.}$$

$$\text{(ii) Total energy consumption} = \frac{15.44}{0.70}$$
$$= 22.06 \text{ kW-hr.}$$

$$\text{(iii) } \left.\begin{array}{r}\text{Specific energy}\\ \text{consumption}\end{array}\right\} = \frac{\text{the total energy consumption in watt} - \text{hours}}{\text{the weight of the train in tons} \times \text{the distance of the run in km}}$$
$$= \frac{22.06 \times 1000}{100 \times 3}$$
$$= 73.53 \text{ W-h/ton-km.}$$

Example 10.24: An electric train has quadrilateral speed–time curve as follows:

1. Uniform acceleration from rest 2 kmphps for 30 s.
2. Coasting for 40 s.
3. Braking period of 25 s.

The train is moving a uniform down gradient of 1% and the tractive resistance of 50 N/ton. The rotational resistance is 10% of the dead weight, the duration of the stop is 20 s and the overall efficiency of the transmission the gear and the motor as 80%. Calculate its schedule speed and specific energy consumption.

Solution:

Acceleration (α) = 2 kmphps.

Acceleration period (t_1) = 30 s.

Gradient (G) = 1%.

The tractive of resistance (r) = 50 N/ton.

The duration of stop = 20 s.

Overall efficiency (η) = 80%.

Maximum speed $V_m = \alpha t_1$

$= 2 \times 30 = 60$ kmph.

Let the coasting retardation be β_c:

Tractive effort:

$F_t = 277.8\ W_c\ (-\beta_c) - 98.1 \times WG + Wr$

$0 = -277.8 \times 1.1\ W \beta_c - 98.1 \times W \times 1 + 50\ W$

$$\beta_c = \frac{-48.1W}{305.58}$$

$\beta_c = -\ 0.157$ kmphps

$V_2 = V_m - \beta_c t_2$

$= 60 - (-0.517 \times 40)$

$= 66.28$ kmph.

$$\text{The distance travelled, } D = \frac{V_1 t_1}{7,200} + \frac{(V_m + V_2)\,t_2}{7,200} + \frac{V_2 t_3}{7,200}$$

$$= \frac{60 \times 30}{7,200} + \frac{(60 + 66.28)}{7,200} \times 40 + \frac{66.28 \times 25}{7,200}$$

$= 0.25 + 0.7 + 0.23$

$= 1.18$ km.

Schedule time, $T_s = t_1 + t_2 + t_3 +$ stop duration

$= 30 + 40 + 25 + 20$

$= 115$ s.

$$\text{Schedule speed, } V_s = \frac{3,600 \times D}{T_s}$$

$$= \frac{3,600 \times 1.18}{115}$$

$= 36.939$ kmph.

The specific energy output:

$$= \frac{0.01072 V_m^2}{D} \times \frac{W_e}{W} + 0.2778(98.1\,G + r)\frac{D_1}{D}$$

$$= \frac{0.01072 \times (60)^2}{1.18} \times 1.1 + 0.2778(98.1 \times 1 + 50) \times \frac{0.25}{1.18}$$

$$= 35.975 + 8.716$$

$$= 44.69 \text{ W-hr/ton-km.}$$

The specific energy consumption $= \frac{44.69}{0.8} = 55.86$ W-hr/ton-km.

Example 10.25: The schedule speed of a electric train is 40 kmph. The distance between two stations is 3 km with each stop is of 30 s duration. Assuming the acceleration and the retardation to be 2 and 3 kmphps, respectively. The dead weight of the train is 20 ton. Assume the rotational inertia is 10% to the dead weight and the track resistance is 40 N/ton. Calculate:

1. The maximum speed.
2. The maximum power output from driving axles.
3. The specific energy consumption is watt-hours per ton-km. The overall efficiency is 80%, assume simplified speed–time curve.

Solution:

Schedule speed V_s = 40 kmph.

The distance between the two stations (D) = 3 km.

The duration of stop = 30 s.

Acceleration (α) = 2 kmphps.

Retardation (β) = 3 kmphps.

The dead weight of the train (w) = 20 ton.

The track resistance (r) = 40 N/ton.

The overall efficiency (η) = 80%.

The schedule time of run $T_s = \dfrac{3,600 \times D}{V_s}$

$$= \frac{3,600 \times 3}{40} = 270\,\text{s}.$$

The actual time of run, $T = 270 - 30$

$$= 240\ \text{s}.$$

(i) The maximum speed, $V_m = \dfrac{T}{2X} - \sqrt{\dfrac{T^2}{4X^2} - \dfrac{3,600D}{X}}$,

where:

$$X = \frac{1}{2\alpha} + \frac{1}{2\beta}$$

$$= \frac{1}{2 \times 2} + \frac{1}{2 \times 3} = 0.416.$$

$$V_m = \frac{240}{2 \times 0.416} - \sqrt{\frac{(240)^2}{4 \times (0.416)^2} - \frac{3,600 \times 3}{0.416}}$$

$$= 288.46 - \sqrt{(83,210.059) - (25,961.538)}$$

$$= 49.193\ \text{kmph}.$$

(ii) The acceleration time, $t_1 = \dfrac{V_m}{\alpha}$

$$= \frac{49.193}{2}$$

$$= 24.59\ \text{s}.$$

The duration of braking, $t_3 = \dfrac{V_m}{\beta} = \dfrac{49.193}{3} = 16.397\,\text{s}.$

The free-running time $t_2 = \text{T} - (t_1 + t_3)$

$$= 240 - (24.59 + 16.397)$$

$$= 199.012\ \text{s}.$$

The tractive effort during acceleration:

$F_t = 277.8W_e \times \alpha + Wr$

= 277.8 × 1.1 × 20 × 2 + 20 × 40

= 13,023.2 N.

$$\text{The maximum power output} = \frac{F_t V_m}{3,600}$$

$$= \frac{13,023.2 \times 49.193}{3,600}.$$

The maximum power output = 177.958 kW.

(iii) The distance travelled during braking:

$$= \frac{1}{2}\frac{V_m \times t_3}{3,600}$$

$$= \frac{1}{2} \times \frac{49.193 \times 16.397}{3,600}$$

$$= 0.112 \text{ km.}$$

The distance travelled with power is on:

D_1 = 3 – 0.112

= 2.88 km.

The specific energy output:

$$= \frac{0.01072V^2_{\mathrm{m}}}{D} \times \frac{W_e}{W} + 0.2778r\frac{D_1}{D}$$

$$= \left[\frac{0.01072 \times (49.193)^2}{3} \times 1.1\right] + \left[0.2778 \times 40 \times \frac{2.88}{3}\right]$$

$$= 9.512 + 10.667$$

$$= 20.179 \text{ W-hr/ton-km.}$$

$$\text{The specific energy consumption } = \frac{20.179}{\text{efficiency}} = \frac{20.179}{0.8}$$

$$= 25.244 \text{ W-hr/ton-km.}$$

CALCULATION OF ENERGY RETURNED TO THE SUPPLY DURING REGENERATIVE BRAKING

When the train is accelerating, it acquires kinetic energy corresponding to that speed. During the coasting period, some of the kinetic energy is wasted, to propel the train against the friction and windage resistance.

While the train is moving on the down gradients or level track, the KE acquired by the rotating parts is converted into the electrical energy, which is fed back to the supply system. The amount of energy fed back to the system is depending on the following factors.

1. The initial and final speeds during the regenerative braking.
2. The train resistance and the gradient of the track.
3. The efficiency of the system.

Consider the initial and final speeds of the train during regenerative braking are V_1 and V_2 in KMPH, and the effective weight of the train is W_e tons.

$$\text{Thus, the mass of the train, } m = \frac{W_e}{g} \text{ tons/(m/s}^2\text{)}$$

$$= \frac{1000W_e}{9.81} \text{kg/(m/s}^2\text{).}$$

The speed of the train = V_1 kmph

$$= \frac{V_1 \times 1{,}000}{3{,}600} \text{ m/s.}$$

The kinetic energy stored by the train at a speed of V_1 kmph:

$$= \frac{1}{2} \times mv^2 \text{J}$$

$$= \frac{1}{2} \times \frac{1{,}000 W_e}{9.81} \times \left[\frac{v_1 \times 1{,}000}{3{,}600}\right]^2 \frac{\text{kg}}{\text{m/s}^2} \times \frac{\text{m}}{\text{s}}$$

$$= \frac{1}{2} \times \frac{1{,}000 W_e}{9.81} \times \left[\frac{v_1 \times 1{,}000}{3{,}600}\right]^2 \times 9.81 \text{N-m} (\text{or}) W\text{-sec}$$

[1 kg-m = 9.81 N-m]

$$= \frac{1}{2} \times \frac{1{,}000 W_e}{9.81} \times \left[\frac{v_1 \times 1{,}000}{3{,}600}\right]^2 \times 9.81 \times \frac{1}{3{,}600} \text{W-h}$$

[1 W-Sec = 1/3,600 W-h]

$$= 0.01072 V_1^2 W_e \text{ W-h}$$

$$= 0.01072 V_1^2 \left(\frac{W_e}{w}\right) \text{W-h/ton.}$$

Thus, the kinetic energy at speed V_2 kmph:

$$= 0.01072 V_2^2 \left(\frac{W_e}{w}\right) \text{W-h/ton.}$$

Therefore, the energy available during the regeneration:

$$= 0.01072 \left(\frac{W_e}{w}\right) \times \left(V_2^2 - V_1^2\right) \text{W-h/ton.}$$

If D is the distance in km covered during the regenerative braking, then the energy fed back to the supply during the braking while the train is moving on down gradient:

$$= WD \times \frac{G}{100} \text{ ton-km}$$
$$= (1,000 \times w) \times (1,000 \times D) \times \frac{G}{100}$$
$$= WDG \times 10^4 \text{ kg-m (or) N-sec}^2$$
$$= 9.81 \times WDG \times 10^4 \text{ N-m (or) W-sec}$$
$$= \frac{9.81 \times WDG \times 10^4}{3,600} \text{ W-h}$$
$$= 27.25\ WDG \text{ W-h}$$
$$= 27.25\ DG \text{ W-h/ton.}$$

If r is the train resistance in N/ton, then the energy lost to overcome the resistance to the motion and friction, windage losses:

$$= WrD \text{ N-km}$$
$$= WrD \times 1,000 \text{ N-m (or) w-sec}$$
$$= \frac{WrD \times 1,000}{3,600} \text{W-h}$$
$$= 0.2778\ WrD \text{ W-h (or) } 0.2778\ rD \frac{\text{W-h}}{\text{ton}}.$$

Hence, the total energy available during regeneration:

$$= \left[0.01072\left(\frac{W_e}{W}\right)(V_1^2 - V_2^2) + 27.25\ DG - 0.2778 rD\right] \text{W-h/ton.}$$

The energy returned to the supply system:

$$= \left[0.01072\left(\frac{W_e}{W}\right)(V_1^2 - V_2^2) + 27.25\ DG - 0.2778rD\right] \times \eta\ \boldsymbol{W}\text{-h/ton,}$$

where η is the efficiency of the system.

Advantages of regenerative breaking

1. In regenerative breaking, a part of the energy stored by the rotating parts is converted into the electrical energy and is fed back to the supply. This will lead to the minimum consumption of energy, thereby saving the operating cost.
2. High breaking retardation can be obtained during regenerative breaking.
3. Time taken to bring the vehicle to rest is less compared to the other breakings; so that, the running time of the vehicle is considerably reduced.
4. The wear on the brake shoes and tyre is reduced, which increases the life of brake shoe and tyre.

Disadvantages

In addition to the above advantages, this method suffers from the following disadvantages.

1. In addition to the regenerative breaking, to bring the vehicle to standstill, mechanical breaking is to be employed.
2. In case of DC traction, additional equipment is to be employed for regenerative breaking, which increases the cost and sometimes, substation are equipped with mercury arc rectifiers to convert AC to DC supply.
3. The electrical energy returned to the supply will cause the operation of substations complicated.

Example 10.26: A 450-ton train travels down gradient of 1 in 75 for 110 s during which its speed is reduced from 70 to 55 kmph. By the regenerative braking, determine the energy returned to the lines if the reactive resistance is 4.5 kg/ton and the allowance for the rotational inertia is 7% and the overall efficiency of the motor is 80%.

Solution:

Accelerating weight $W_a = 1.075 \times 450$

$= 483.75$ ton.

Track resistance $r = 9.81 \times 4.5$

$= 44.145$ N-m/ton

$= 9,904.782$ W-hr

$= 9.904$ kW-hr.

Factiva effort required during retardation:

$= Wr - 98.1\ \mathrm{W}G$

$= 450 \times 44.45 - 98.1 \times 450 \times 4/3$

$= 19,865.25 - 58,860$

$= -38,994.75$ N.

The distance travelled during the retardation period:

$$= \frac{V_1 + V_2}{2} \times \frac{1,000}{3,600} \times \mathrm{T}$$

$$= \frac{70 + 55}{2} \times \frac{1,000}{3,600} \times 110$$

$$= 1,909.73 \text{ m.}$$

As the train is moving in downward gradient, so that the tractive effort will provide additional energy to the system. The energy available when the train moves over a gradient is given as:

$$= \frac{38,994.75 \times 1,909.73}{1,000 \times 3,600}.$$

$$\text{Gradient } G = \frac{1}{75} \times 100$$

$$= \frac{4}{3} \times 100.$$

The period of regeneration = 110 s.

Overall efficiency (η) = 80%.

The kinetic energy of the train at a speed of 70 kmph is:

$$= 0.01092 V_1^2 \; Wa$$
$$= 0.01092 \times (70)^2 \times (483.75)$$
$$= 25,884.495 \text{ W-hr.}$$

The kinetic energy of the train at the speed of 55 kmph is:

$$= 0.01092 V_2^2 \; Wa$$
$$= 0.01092 \times (55)^2 \times (483.75)$$
$$= 15,979.713 \text{ W-hr.}$$

The energy available due to the retardation by the regenerative braking:

= 2,588.495 – 15,979.713

= 20.68 kW-hr.

The energy returned to the supply system:

= 0.8 × total energy available

= 0.8 × (20.68+9.904)

= 24.467 kW-hr.