

Tutorial II, B.Tech. Sem II
Linear Differential Equations with Constant Coefficients, Cauchy Euler form

Solve the following differential equations

1. $(D^2+a^2)y = \operatorname{cosec} x$; **Ans:** $y = c_1 \cos ax + c_2 \sin ax + (1/a^2) \sin ax \log \sin ax - (x/a) \operatorname{cosec} ax$
2. $(D^3 - 1)y = (e^x + 1)^2$;
Ans: $y = c_1 e^x + e^{-\frac{1}{2}x} [c_2 \cos(\sqrt{3}x/2) + c_3 \sin(\sqrt{3}x/2)] - 1 + (1/7) e^{2x} + (2/3) x e^x$
3. $(D^2 - 2kD + k^2)y = e^{kx}$;
Ans: $y = (c_1 + c_2 x)e^{kx} + (x^2/2) e^{kx}$
4. $\frac{d^3 y}{dx^3} + 2 \frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 8y = x$, **Ans:** $y = c_1 e^{-2x} + c_2 \cos(2x + c_3) + \frac{1}{16}(2x - 1)$
5. $(D^3 - 5D^2 + 7D - 3)y = e^{2x} \cos hx$; **Ans:** $y = (c_1 + c_2 x) e^x + c_3 e^{2x} - (1/8)x^2 e^x + (1/8)x e^{3x}$
6. $(D^3 + D^2 - D - 1)y = \cos 2x$; **Ans:** $y = c_1 e^x + (c_2 + c_3 x) e^{-x} - (1/25)(2 \sin 2x + \cos 2x)$
7. $(D^4 - 1)y = \sin 2x$; **Ans:** $y = c_1 e^x + c_2 e^{-x} + c_3 \cos x + c_4 \sin x + (1/15) \sin 2x$
8. $(D^2 - 4)y = \cos^2 x$; **Ans:** $y = c_1 e^{2x} + c_2 e^{-2x} - (1/16)(2 + \cos 2x)$
9. $(D^3 + 8)y = x^4 + 2x + 1$; **Ans:** $y = c_1 e^{-2x} + e^x (c_2 \cos x\sqrt{3} + c_3 \sin x\sqrt{3}) + (1/8)(x^4 - x + 1)$
10. $(D^4 - 2D^3 + 5D^2 - 8D + 4)y = x^2$;
Ans: $y = (c_1 + c_2 x) e^x + c_3 \cos 2x + c_4 \sin 2x + (1/4)(x^2 + 4x + 11/2)$
11. $(D^2 - 1)y = \cos hx \cos x$; **Ans:** $y = c_1 e^x + c_2 e^{-x} + (2/5) \sin hx \sin x - (1/5) \cos hx \cos x$
12. $(D^3 + 1)y = e^{2x} \sin x + e^{x/2} \sin(x\sqrt{3}/2)$;
Ans: $y = c_1 e^{-x} + e^{x/2} [c_2 \cos(x\sqrt{3}/2) + c_3 \sin(x\sqrt{3}/2)] - (x/6) e^{x/2} [\sin(x\sqrt{3}/2) + \sqrt{3} \cos(x\sqrt{3}/2)]$
13. $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$; **Ans:** $y = (c_1 + c_2 x)e^{2x} + e^{2x} (3 \sin 2x - 2x^2 \sin 2x - 4x \cos 2x)$
14. $(D^4 + 2D^3 - 3D^2)y = 3e^{2x} + 4 \sin x$;
Ans: $y = c_1 + c_2 x + c_3 e^x + c_4 e^{-3x} + (3/20) e^{2x} + (2/5)(2 \sin x + \cos x)$
15. $\frac{d^4 y}{dx^4} + \frac{d^2 y}{dx^2} + y = a x^2 + b e^{-x} \sin 2x$;
Ans: $y = c_1 e^{-x/2} \cos(x\sqrt{3}/2 + c_2) + c_3 e^{x/2} \cos(x\sqrt{3}/2 + c_4) + a x^2 - 2a - (1/481) b e^{-x} [9 \sin 2x + 20 \cos 2x]$
16. $\frac{d^2 y}{dx^2} - y = 1$, $y(0) = 0$ and $y(-\infty) \rightarrow \text{finite}$; **Ans:** $y = e^x - 1$
17. $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 10y + 37 \sin 3x = 0$, $y(0) = 3$ and $dy/dx = 0$ at 0 ; Find y at $\pi/2$ **Ans:** $y = 1$
18. $\frac{d^2 x}{dt^2} + 4x = a \sin t \cos t$; **Ans:** $x = c_1 \cos 2t + c_2 \sin 2t - \frac{1}{8} a t \cos 2t$
19. $(D^2 - 2D + 1)y = x^2 e^{3x}$; **Ans:** $y = (c_1 + c_2 x) e^x + \frac{e^{3x}}{4} (x^3 - 2x + \frac{3}{2})$
20. $x^3 \frac{d^3 y}{dx^3} + 3x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = x + \log x$; **Ans:** $y = c_1/x + c_2 \sqrt{x} \cos(\frac{\sqrt{3}}{2} \log x + c_3) + \frac{1}{2} x + \log x$

$$21. y \frac{d^2 y}{dx^2} = \left(\frac{dy}{dx}\right)^2 \left[1 - \cos y \frac{dy}{dx} + y \frac{dy}{dx} \sin y\right]$$

$$22. x \frac{d^2 y}{dx^2} - (1-x) \frac{dy}{dx} - y = e^x$$

$$23. x \frac{d^2 y}{dx^2} - (1-x) \frac{dy}{dx} - y = e^x$$

R. K. SOMI