Department of Mathematics
Faculty of Engineering & Technology
VBS Purvanchal University, Jaunpur

Subject: Discrete Structure and Theory of Logic (KCS-303)
Syllabus

	DETAILED SYLLABUS	3-0-0
Unit	Topic	Proposed Lecture
I	**Relations:** Definition, Operations on relations, Properties of relations, Composite Relations, Equality of relations, Recursive definition of relation, Order of relations. **Functions:** Definition, Classification of functions, Operations on functions, Recursively defined functions. Growth of Functions. **Natural Numbers:** Introduction, Mathematical Induction, Variants of Induction, Induction with Nonzero Base cases. Proof Methods, Proof by counter – example, Proof by contradiction.	**08**
II	**Algebraic Structures:** Definition, Groups, Subgroups and order, Cyclic Groups, Cosets, Lagrange's theorem, Normal Subgroups, Permutation and Symmetric groups, Group Homomorphisms, Definition and elementary properties of Rings and Fields.	**08**
III	**Lattices:** Definition, Properties of lattices – Bounded, Complemented, Modular and Complete lattice. Boolean Algebra: Introduction, Axioms and Theorems of Boolean algebra, Algebraic manipulation of Boolean expressions. Simplification of Boolean Functions, Karnaugh maps, Logic gates, Digital circuits and Boolean algebra.	**08**
IV	**Propositional Logic:** Proposition, well formed formula, Truth tables, Tautology, Satisfiability, Contradiction, Algebra of proposition, Theory of Inference. **Predicate Logic:** First order predicate, well formed formula of predicate, quantifiers, Inference theory of predicate logic.	**08**
V	**Recurrence Relation & Generating function: Recursive** definition of functions, Recursive algorithms, Method of solving recurrences. **Combinatorics:** Introduction, Counting Techniques, Pigeonhole Principle **Number Theory:** Introduction, Basic Properties, Divisibility Theory, Congruences, Applications of Congruences.	**08**

Text books:

1. Koshy, Discrete Structures, Elsevier Pub. 2008 Kenneth H. Rosen, Discrete Mathematics and Its Applications, 6/e, McGraw-Hill, 2006.
2. B. Kolman, R.C. Busby, and S.C. Ross, Discrete Mathematical Structures, 5/e, Prentice Hall, 2004.
3. E.R. Scheinerman, Mathematics: A Discrete Introduction, Brooks/Cole, 2000
4. R.P. Grimaldi, Discrete and Combinatorial Mathematics, 5/e, Addison Wesley, 2004
5. Liptschutz, Seymour, "Discrete Mathematics", McGraw Hill.
6. Trembley, J.P & R. Manohar, "Discrete Mathematical Structure with Application to ComputerScience", McGraw Hill
7. Narsingh, "Graph Theory With application to Engineering and Computer.Science.", PHI.
8. Krishnamurthy, V., "Combinatorics Theory & Application", East-West Press Pvt. Ltd., New Delhi

Question Bank

UNIT - I

1. Let $A = \{a, b, c\}$ and the relation R be defined on A as follows:
   $R = \{ (a, a), (b, c), (a, b) \}$. Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive. **Ans:** 3.
2. Let D be the domain of real valued function f defined by $f(x) = \sqrt{25 - x^2}$ then, write D. **Ans :** $D = [-5,5]$,
3. Show that a set A with 3 elements has 2^6 symmetric relations on A. **Hint**: $2^{n(n+1)/2}$
4. Is $g = \{(1, 1), (2, 3, (3, 5), (4, 7)\}$ a function? If g is described by g(x) = αx +βy, then what value should be assigned to α and β? **Ans** : $\alpha = 2, \beta = -1$.
5. If R ={ (a, a^3): a is a prime number less than 5 } be a relation. Find the range of R.
   Ans : {8,27}.
6. Let R be the equivalence relation in the set A = {0,1,2,3,4,5} given by
   R = {(a,b): 2 divides (a - b)}. Write the equivalence class [0].
   Ans: equivalence class of [0] = [2,4].
7. If $R = \{(x, y): x + 2y = 8\}$ is a relation on N, then write the range of R.

8. **Ans**: Range={3, 2, 1}.

9. If $A = \{1,2\}, B = \{2,3,4\}, C = \{4,5\}$, then find: $A \times (B \cap C)$.

10. If $P = \{1,3\}, Q = \{2,3,5\}$ find the number of relations from P to Q. **Ans**: 64.

11. If the ordered Pairs $(x - 1, y + 3)$ and $(2, x + 4)$ are equal, find x and y.

Ans :$x = 3, y = 4$.

12. If a and b are any two elements of group G then prove : $(a^*b)^{-1} = b^{-1} * a^{-1}$.

13. If f : A → B is one-one onto mapping, then prove that f^{-1}: B → A will be one-one onto mapping.

14. Let R be a relation on the set of natural numbers N, as R = {(x, y) : x, y ∈ N, 3x + y = 19}. Find the domain and range of R. Verify whether R is reflexive.

15. Show that the relation R on the set Z of integers given by R = {(a, b) : 3 divides a – b}, is an equivalence relation.

16. Let $A = \{ a_1, a_2, a_3 \}, B = \{ b_1, b_2, b_3, b_4 \}$. Find the matrix relation.

17. Define injective, surjective and bijective function.

18. Find the power set of each of these sets, where a and b are distinct elements

$(i) \{a, \{b\}\}, (ii) \{1, \emptyset, \{\emptyset\}\}$.

19. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

20.. The following relation on A = {1, 2, 3, 4}. Determine whether the following: (a) R = {(1, 3),(3,1), (1, 1), (1, 2), (3, 3), (4, 4)}. (b) $R = A \times A$ is an equivalence relation or not.

21.. Let R be a binary relation defined as R = { (a,b) ∈ $R^2 : (a, b) \leq 3$} determine whether R is reflexive, symmetric, anti symmetric and transitive and how many distinct binary relation are there on finite set.

22. Let A = { 1,2,3,4,5,6 } and let R be the relation defined by x divides y written as x/y :

(a) Write R as a set of ordered pairs.

(b) Draw its directed graph.

(c) Find R^{-1}.

23. If f : A → B, g : B → C are invertible functions, then show that gof : A → C is invertible and $(gof)^{-1} = f^{-1} o\, g^{-1}$.

24. State whether the function f: N→ N given by f(x) = 5x is injective, surjective or both.

25. Given a non empty set X, consider P(X) which is the set of all subsets of X. Define the relation R in P(X) as follows: For subsets A, B in P(X), A R B if and only if A⊂B. Is R an equivalence relation on P(X)?

26. Write short notes on : a. Equivalence relation b. Composition of relation.

27. Show that the relation R in the set {1, 2, 3} given by R= {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor transitive.

28. Let X = {1, 2, 3,....., 7} and R = {(x, y)|(x – y) is divisible by 3}. Is R equivalence relation. Draw the digraph of R.

29. Determine whether each of these functions is a bijective from R to R:

(a) $f(x) = x^2 + 1$ (b) $f(x) = x^3$ (c) $f(x) = (x^2 + 1)/(x^2 + 2)$.

30. Let R ={ (1,2), (2,3), (3,1) } and A = { 1,2,3 } , find the reflexive , symmetric and transitive closure of R ,using (i) Composition of relation R. (ii) Composition of matrix relation R. (iii) Graphical representation of R.

31. Show that the function f and g both of which are from N × N to N given by $f(x, y) = x + y$ and $g(x, y) = xy$ are onto but not one-one.

32. The composition of any function with the identity function is the function itself *i.e.* $(f\ o\ I_A)(x) = (I_B o\ f)(x) = f(x)$.

33. Show that $\sqrt{3}$ is not a rational number.

34. By the first principle of mathematical induction, prove that: $3^{2n+1} + (-1)^n 2 \equiv (mod\ 5)$.

35. Prove that $n^3 + 2n$ is divisible by 3 using principle of mathematical induction, where n is natural number.

UNIT -II

1. Define : (a) Groupoid (b) Semigroup (c) Monoid.
2. Define Group and write its properties.
3. Let Z be the group of integers with binary operation * defined by a * b = a + b – 2, for all a, b ∈ Z. Find the identity element of the group (Z,*).
4. Define order of finite and infinite Group.
5. Define Homomorphism of Group.
6. Show that every cyclic group is abelian.
7. Define a binary operation? Give an example of a binary operation which is not associative.
8. Let S= $\{e, a, b, c\}$ be a group with binary operation $*$,then complete the following group table.(Explanation table says that a $*$ b = c).

*	e	a	b	c
e	e	a	b	c
a	a		c	
b	b			
c	c			

9. Let S be a set of all real numbers except -1. Define $*$ on S by the rule $a * \mathrm{b} = \mathrm{a} + \mathrm{b} + \mathrm{ab}\ \forall\ a, b \in S$. Show that (S,*) is a group.
10. Prove that the intersection of any subgroup of a group (G,$*$) .
11. Let G be a group. If $a, b \in G$ such that $a^4 = e$,the identity element of G and $ab = ba^2$,prove that $a = e$.

12. Show that the binary operation * defined on $(R,*)$ where $x * y = max\,(x, y)$ is associative.

13. If the permutation of the elements of $\{1, 2, 3, 4, 5\}$ are given by $a = (1\ 2\ 3)(4\ 5), b = (1)\ (2)\ (3)\ (4\ 5),\ c = (1\ 5\ 2\ 4)\ (3)$. Find the value of x, if $ax = \text{b}$. And also prove that the set Z_4 = (0, 1, 2, 3) is a commutative ring with respect to the binary modulo operation $+_4$ and $*_4$.

14. Show that every group of order 3 is cyclic.

15. Obtain all distinct left cosets of {(0), (3)} in the group (Z_6, $+_6$) and find their union.

16.. Let G be the set of all non-zero real number and let $a * b = ab/2$. Show that $(G,*)$ be an abelian group.

17. Show that the set of integers forms an abelian group under addition.

18. Show that the cube root of unity is an abelian group under multiplication.

19.. Let $G = \{1, -1, i, -i\}$ with the binary operation multiplication be an algebraic structure, where $i = \sqrt{-1}$. Determine whether G is an abelian or not.

20. Let Z be the set of integers, show that the operation * on Z defined by $a * b = a + b$ for all $a, b \in Z$ satisfies the closure property, associative law and the commutative law. Find the identity element. What is the inverse of an integer?

21. Show that $[((p \vee q) \rightarrow r) \wedge (\sim p)] \rightarrow (q \wedge r)$ is tautology or contradiction.

22. Show that the matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

form a multiplicative abelian group.

23. Let Q be the set of positive rational numbers which can be expressed in the form $2^a 3^b$, where a and b are integers, Prove that the algebraic structure $(Q,$ $.)$ is a group where is multiplication operator.

24.Find the order of every element in the multiplicative group

$$G = \{a, a^2, a^3, a^4, a^5, a^6 = e\}$$

25.If G be an abelian group with identity e, then prove that all elements x of G satisfying the equation $x^2 = e$ from a sub-group H of G.

UNIT – III

1. Define Partially Ordered sets.
2. Draw the Hasse diagram of D_{30}.
3. Prove that a lattice with 5 elements is not a boolean algebra.
4. Define minimal and maximal element in Posets.
5. Draw the Hasse diagram of [P (a, b, c), ⊆] (Note : '⊆' stands for subset). Find greatest element, least element, minimal element and maximal element.
6. Distinguish between Bounded lattice and Complemented lattice.
7. Prove that in a distributive lattice ,if an element has complement then this complement is unique.
8. Prove that $(i)\ (a+b)' = a'.b'$ $(ii)\ (a.b)' = a' + b'$.
9. Obtain the equivalent expression for $[(x.y)(z' + xy')]$.
10. In a lattice if a ≤ b ≤ c, then show that

    (a) a ∨ b = b ∧ c.

    (b) (a ∨ b) ∨ (b ∧ c) = (a ∨ b) ∧ (a ∨ c) = b.
11. (a) Prove that every finite subset of a lattice has an LUB and a GLB.

    (a) (b) Give an example of a lattice which is a modular but not a distributive.

12.Convert the poset of divisors of 36 into a totally ordered set in two ways.

13. Explain modular lattice, distribute lattice and bounded lattice with example and diagram.

14. Find the Sum-Of-Products and Product-Of-sum expansion of the Boolean function F(x, y, z) = (x + y) z′.

Ans: $SOP = xyz' + z'x'y + xy'z'$, $POS = (x' + y' + z).(x' + y + z).(x + y' + z)$.

15. Minimize the following Boolean function :

$F(a, b, c, d) = \sum m(0,1,2,5.7,8,9,10,13,15)$. **Ans :** BD+C'D+B'D'

16. Minimize the following Boolean function :

$$F(a, b, c, d) = \sum m(0,2,8,10,14) + \sum d(5,15).$$

And design logic circuit usind NAND gate. **Ans:** ACD'+B'D'

UNIT -IV

1. Define the terms with example supporting to each: (a) Proposition (b) Compound proposition (c) Disjunction and Conjunction (d) Conditional and Bi conditional (e) Tautology,Contradiction and Contingency.
2. How can this sentence be translated into a logical expression ? "you can access the internet from campus only if you are a computer science major or are not a freshman. Ans : $p \to (q \vee \neg r)$.
3. Construct the truth table of $(p \vee \neg q) \to (p \wedge q)$.
4. Prove that $(p \vee q) \to (p \wedge q)$ is logically equivalent to $P \leftrightarrow Q$.
5. Prove that : $\neg(p \vee q) \equiv (\neg p \wedge \neg q)$. (De Morgan's Law)
6. Prove that : $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$. (Distributive Law)
7. Negate the statements (i) All integers are greater than 8. (ii) For all real numbers x,if $x > 3$ then $x^2 > 9$.

   Ans :(i) $\sim \forall x \left(p\ (x)\right) \equiv \exists x \left(\sim p(x)\right)$. (ii) $\exists x\ (p\ (x \wedge \sim q\ (x))$.

8. Let p and q be the propositions

   p : It is below freezing.

   q : It is snowing.

   Write these propositions using p and q and logical connectives (including negations).

   a) It is below freezing and snowing.

   b) It is below freezing but not snowing.

   c) It is not below freezing and it is not snowing.

   d) It is either snowing or below freezing (or both).

   e) If it is below freezing, it is also snowing.

   f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing

9. How many rows appear in a truth table for each of these compound propositions?

   a) $p \to \sim p$

   b) $(p \vee \sim r) \wedge (q \vee \sim s)$

   c) $q \vee p \vee \sim s \vee \sim r \vee \sim t \vee u$

   d) $(p \wedge r \wedge t) \leftrightarrow (q \wedge t)$

10. Prove that if n is a positive integer, then n is even if and only if 7n + 4 is even.

11. Show that the premises "If you send me an e-mail message, then I will finish writing the program," If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed." **Ans with Hint**: Use, Contrapositive, Hypothetical syllogism.

12. Show that the following statement is tautology :

$((p \vee q) \wedge (p \to r) \wedge (q \to r)) \to r.$

UNIT – V

1. Define Recurence relation with example.
2. Find the first four terms of each of the following recurrence relation:

   (a) $a_k = 2a_{k-1} + k, \forall\ integers\ \ k \geq 2, a_1 = 1.$ **Ans**: $a_1 = 1, a_2 = 4, a_3 = 4, a_4 = 26.$

   (b) $a_k = k(a_{k-1})^2, \forall\ integers\ \ k \geq 2, a_0 = 1.$ **Ans**: $a_0 = 1, a_1 = 1, a_2 = 2, a_3 = 12.$
3. Find the recurrence relation from $y_n = A\,2^n + B(-3)^n$.

   Ans: $\mathrm{y_{n+2} - y_{n+1} - 6y_n = 0}.$
4. What is the generating function of $\{1,1,1,1,1,\ldots\ldots\}$. Ans : $1/1 - x$.
5. Solve the recurrence relation using generating function:

   $a_n - 7a_{n-1} + 10a_{n-2} = 0$ with $a_0 = 3, a_1 = 3.$ **Ans :** $a_n = 4.2^n - 5^n.$
6. Solve the recurrence relation using Iteration method :

   $a_n = a_{n-1} + 2, n \geq 2, a_1 = 3.$ Ans : $a_n = 3 + (n-1).2$
7. Solve the recurrence relation:

   (a) $a_{n+2} - 5a_{n+1} + 6a_n = 2, a_0 = 1, a_1 = -1.$ **Ans**:$a_n = -2.3^n + 2.2^n + 1.$

   (b) $y_{n+2} - y_{n+1} - 2y_n = n^2.$ **Ans** :$y_n = C_1(-1)^n + C_2.2^n - 1 - n/2 - (1/2)\,n^2.$

   (c) $a_n^2 - 2a_{n-1}^2 = 4, for\ \ n \geq 1\ and\ \ a_0 = 3.$ **Ans :** $a_n = \sqrt{13.2^n - 4}.$
8. State and Prove Pigeonhole Principle.
9. Prove that $(2n)! = 2^n\, n!\{1,3,5, \ldots \ldots ..(2n-1)\}.$
10. A coin tossed 6 times .In how many ways can we obtain 4 heads and 2 tails?

    Ans: 15
11. Find the *gcd* of 595 and 252 and express it in the form $252m + 595n$.

    Ans: $gcd(595,252) = 7.$
12. Find the integers x and y such that $71x - 50y = 1$. **Ans**: $x = 31, y = 44.$
13. If $a \equiv b(mod\ m)$ and c is any integer,then prove that:

    (a) $(a + c) \equiv (b + c)(mod\ m)$

(b) $(a-c) \equiv (b-c)(mod\ m)$

14. The first 9 digits of this book are 81-219-2232. What is the check digit for this book? **Ans:** Check digit $x_{10} = 1$.

15. Find the remainder when the following sum is divisiblw by 4, $1^5 + 2^5 + 3^5 + \cdots \ldots \ldots \ldots .. + 100^5$. **Ans** : zero

16. Use the theory of congruence to prove that for $n \geq 1, 17 \mid (2^{3n+1} + 3.5^{2n+1})$.