Tutorial I
B.Tech. Sem II Mathematics II

1. Define the terms with one example supporting to each: (a) Order and Degree of a differential equation.(b) Linear, semi–linear and nonlinear differential equation.

2. Find the Order and Degree of (i) $y = \sqrt{x}\frac{dy}{dx} + \frac{k}{\frac{dy}{dx}}$, (ii) $k\frac{d^2y}{dx^2} = \left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$

3. Find the differential equation of the circles passing through the origin and having their centres on the x–axis. **Ans:** $2xyy' = y^2 - x^2$

4. Write the order and degree of the differential equation: $x^2(d^2y/dx^2)^3 + y(dy/dx)^4 + y^4 = 0$;

5. Find the differential equation corresponding to (i) $y = ae^{3x} + be^x$. (ii) $x = a\sin(wt + b)$. (iii) $y = Ae^x + Be^{-x} + C$.

6. Show that $Ax^2 + By^2 = 1$ is the solution of $x\{y\frac{d^2y}{dx^2} + (\frac{dy}{dx})^2\} = y\frac{dy}{dx}$.

7. Determine the differential equation whose set of independent solution is $\{e^x, xe^x, x^2e^x\}$. **Ans:** $y''' - 3y'' + 3y' - y = 0$

8. Formulate the differential equation for $y = c(x - c)^2$ **Ans:** $y'^3 = 4y(xy' - 2y)$

9. Which of the following set of functions are L.I. (a). $\{e^x, xe^x, \sinh x\}$. (b). $\{1+x, 1+2x, x^2\}$, (c). $\{\sin 3x, \sin x, \sin^3 x\}$, **Ans:** (a), (b)

10. Solve (i) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$, **Ans:** $y-x = c(1+xy)$; (ii) $\frac{dy}{dx} = \frac{x(2\log x+1)}{\sin y+y\cos y}$, **Ans:** $y\sin y = x^2\log x+c$;
    (iii) $xy\frac{dy}{dx} = y + 2$, where $y(1) = 1$,**Ans:** $e^y = \frac{e}{9}x(y + 2)^2$
    (iv) $\frac{dy}{dx} = \cos(x + y)$, **Ans:** $x + \arcsin(x + y) + \cot(x + y) + c = 0$
    (v) $y(\sqrt{1 - x^2})dy + x(\sqrt{1 - y^2})dx = 0$. **Ans:** $\sqrt{1 - x^2} + \sqrt{1 - y^2} + c = 0$
    (vi) $\frac{dy}{dx} = e^{x+y} + x^2e^{x^3+y}$, **Ans:** $e^x + e^{-y} + \frac{1}{3}e^{x^3} + c = 0$
    (vii) $y'\sin x = y\log y$, $y = 1$ when $x = \pi/2$. **Ans:** $y = 1$
    (viii) $\frac{dy}{dx} = xy + x + y + 1$. **Ans:** $\log(y + 1) = \frac{(x+1)^2}{2} + c$.
    (ix) $\tan y\frac{dy}{dx} = \sin(x + y) + \sin(x - y)$. **Ans:** $\sec y + 2\cos x + c = 0$.
    (x) $\frac{dy}{dx} = \frac{2}{x+2y-3}$ (v-separable).
    (xi) $(x^2 - y^2)dx + 2xydy = 0$. (Homogeneous).
    (xii) $x\frac{dy}{dx} = y(\log y - \log x + 1)$. (Homogeneous).
    (xiii) $ye^{(x/y)}dx = (xe^{(x/y)} + y^2)dy$. (Homogeneous).

11. Integrate $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ and obtain the cubic curve satisfying this equation and passing through origin.

12. Solve (i) $x^2y - x^3\dfrac{dy}{dx} = y^4\cos x$ **Ans**: $x^3y^{-3} = c + 3\sin x$.
(ii) $y(2xy + e^x)dx - e^x dy = 0$, **Ans**: $e^x = y(c - x)y$
(iii) $ydx - xdy + (1 + x^2)dx + x^2\sin y = 0$, **Ans**: $x^2 - y - 1 - x\cos y = cx$
(iv) $(xy^2 + 2x^2y^3)dx + (x^2y - x^3y^2)dy = 0$, **Ans**: $\log\frac{x^2}{y} - \frac{1}{xy} = c$
(v) $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$, **Ans**: $3x^2y^4 + 6xy^2 + 2y^6 = c$
(vi) $(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0$, **Ans**: $4x^{1/2}y^{1/2} - \frac{2}{3}x^{-\frac{3}{2}}y^{\frac{3}{2}} = c$
(vii) $\frac{xdx+ydy}{xdy-ydx} = \sqrt{\frac{a^2-x^2-y^2}{x^2+y^2}}$, **Ans**: $y(1 + x^2) = \arctan x - \frac{\pi}{4}$

13. Show that the current in a coil containing a resistance R, an inductance L, and a constant e.m. f. E at time t is given by $I = \frac{E}{R}(1 - e^{\frac{-Rt}{L}})$.

14. Solve the equation $L\dfrac{di}{dt} + R\,i = E_0\sin\omega t$, where L R and E_0 are constants and discuss the case when t increases indefinitely.

15. A resistance of 100 ohms, an inductance of 0.5 Henry are connected in series with a battery of 20 volts. Find the current in the circuit as a function of time.

16. Solve (i) $\dfrac{dx}{dy} = \frac{y}{x+\sqrt{xy}}$.
(ii) $r\sin\theta - \cos\theta\dfrac{dr}{d\theta} = r^2$, **Ans:** $1 = r(c\cos\theta + \sin\theta)$;
(iii)$dr + (2r\cot\theta + \sin 2\theta)d\theta = 0$.
(iv) $y\log y\dfrac{dx}{dy} + x = \log y$.
(v) $x\log x\dfrac{dy}{dx} + y = 2\log x$.
(vi) $\dfrac{dy}{dx} + xy = y^2e^{x^2/2}\sin x$.**Ans:** $e^{-x^2/2} = (c + \cos x)y$;
(vii) $[y(1 + 1/x) + \cos y]dx + [x + \log x - x\sin y]dy$.**Ans:** $y(x + \log x) + x\cos y = c$;
(viii) $[y(1 + 1/x) + \cos y]dx + [x + \log x - x\sin y]dy$. **Ans:** $y(x + \log x) + x\cos y = c$;
(ix) $(1 + xy)ydx + (1 - xy)xdy = 0$. **Ans:** $\log(x/y) = c + \frac{1}{xy}$;
(x) $x^2ydx - (x^3 + y^3)dy = 0$. **Ans:** $y = c\,e^{\frac{x^3}{3y^3}}$.
(xi) $(y + \frac{1}{3}y^3 + \frac{1}{2}x^2)dx + \frac{1}{4}(x + xy^2)dy = 0$. **Ans:** $3x^4y + x^4y^3 + x^6 = c$.
(xii) $y(x^2y^2 + xy + 1)dx + (x^2y^2 - xy + 1)xdy = 0$. **Ans:** $xy\log(x/y) - \frac{1}{xy} = c$;