Tutorial- 07 (B.Tech Sem-II)
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1.Define the terms with one example of each: (a) Bounded sequence, (b) Convergent sequence, (c) Subsequence, (d) Cauchy sequence, (e) Divergent sequence, (f) Cauchy's First theorem on limits, (g) Cauchy convergence criterion, (h) Alternating series, (i) Alternating series test(Leibnitz test).

2. Check the convergence of the following sequences. Find also its limit, if it exists

(a)$\sqrt{n+1}-\sqrt{n}$, n$\in \mathbb{N}$; (b) $\frac{1}{n^p}$, $p>0$; (c) $S_1=\frac{1}{2}$, $S_{n+1}=\frac{2S_n+1}{3}$, n$\in \mathbb{N}$; (d) $S_1=\sqrt{2}$, $S_{n+1}=\sqrt{2S_n}$, n$\in \mathbb{N}$; (e) $\frac{1}{n}(1+2^{1/2}+3^{1/3}+4^{1/4}+\cdots+n^{1/n})$; (f)$\left\{\frac{(3n)!}{(n!)^3}\right\}^{1/n}$ n$\in \mathbb{N}$;

(g) $S_1=1$, $S_{n+1}=\frac{3S_n+4}{2S_n+3}$; (h) $\left[\left(\frac{2}{1}\right)\left(\frac{3}{2}\right)^2\left(\frac{4}{3}\right)^3\cdots\left(\frac{n+1}{n}\right)^n\right]^{1/n}$;(i)$1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}$

Ans: (a) 0, (b) 0, (c) 1, (d) 2, (e) 0, (f) 27, (g)$\sqrt{2}$, (h) e, (i) Does not converge.

3. Check the convergence of the following series using general principle of convergence:

(a) $\sum\frac{1}{4^n}$; (b) $\log\left(\frac{n+1}{n}\right)$, (c) $\sum\frac{1}{n}$; **Ans: :** (a) Convergent, (b) Divergent, (c) Divergent

4. Prove that $\sum\frac{1}{n^p}$ is convergent if p>1 and divergent if p≤1.

5.Test the convergence of the following series:

(I)$2+\frac{3}{2}x+\frac{4}{3}x^2+\frac{5}{4}x^3+\cdots\ldots\ldots, x>0$; (II) $\sum\{(n^3+1)^3-1\}$; (III) $\sum\frac{n^p}{(1+n)^q}$;

(IV) $\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+\cdots$. ; (V) $x+\frac{3}{5}x^2+\frac{8}{10}x^3+\frac{15}{17}x^4+\cdots$..

(VI) $\sum\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$;(VII) $\sum\frac{1}{(n\log n)^p}$, $n\geq 2$ (VIII) $\sum x^n\log(nx)$;

(IX) $1^P+\left(\frac{1}{2}\right)^P+\left(\frac{1.3}{2.4}\right)^P+\left(\frac{1.3.5}{2.4.6}\right)^P+\cdots\ldots$; (X) $\frac{1^2}{2^2}+\frac{1^2.3^2}{2^2.4^2}+\frac{1^2.3^25^2}{2^2.4^26^2}+\ldots\ldots$

Ans:(I) Con. If x<1 and div. If x≥1. (II) Con.(III) con. If p-q+1<0 and div. If p-q+1≥0; (IV) Con. (V) Con If x<1 and div If x≥1, (VI) Conv, (VII)con if p>1 and div if p≤1, (VIII) Con. If x<1 and div. If x≥1, (IX) Con if p>2 and div if p≤2, (X) Div

6. Apply Cauchy Integral test on the series: (a) $\sum\frac{1}{n(\log n)^p}$, $n\geq 2$; Ans: Con if p>1 and div if 0≤p≤1; (b) $\sum\frac{1}{n\sqrt{n^2-1}}$, Ans: Conergent

7.Test the Convergence\ Abs. Convergence and Cond. Convergence of the following series:

(a)$\frac{1^2}{2^2}-\frac{1^2.3^2}{2^2.4^2}+\frac{1^2.3^25^2}{2^2.4^26^2}$-...... (b) $\sum(-1)^{n-1}\frac{n^2}{(n+1)!}$; (c) $\sum(-1)^n\frac{\sin n\alpha}{n^3}$, $\alpha\in\mathbb{R}$

Ans: (a) Cond. Conv.; (b) Abs. Con. ; (c) Convergence.