Tutorial- 5 B. Tech. Math-II 5 April 2024 (Laplace and Inverse Laplace Transforms)

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Q. 1 Find the Laplace transform of the following functions:
(i) $\sin h^3 3t$ **(ii)** $\sin h3t\cos 7t$ **(iii)** $t^2 e^{-3t}\sin 5t$ **(iv)** $e^{-4t}\sin 3t\sin 5t$

Q. 2 Find the Laplace transform of the functions
(i) $f(t)=|t-1|+|t+1|,\ t\geq 0$ **(ii)** $f(t)=\begin{cases}\sin(t-\pi/3), & t>\pi/3\\ 0 & , t<\pi/3\end{cases}$ (iii) $\sin t\cos t\log t\,\delta(t-\pi)$ where, $\delta(t-\pi)$ is Unit Impulse function.

Q.3 Find the Laplace transform of the functions
(i) $f(t)=\dfrac{\cos\sqrt{t}}{\sqrt{t}}$ **(ii)** $f(t)=\dfrac{\sin at}{t}$ where, a is constant. Does the Laplace transform of $f(t)=\dfrac{\cos at}{t}$ exist'

Q. 4 By using the Laplace transform evaluate the following integrals
(i) $\int_0^\infty \frac{\sin t}{t}dt$ **(ii)** $\int_0^\infty t^3 e^{-t}\sin t\,dt=0$ **(iii)** $\int_0^\infty \frac{e^{-t}\sin^2 t}{t}dt$

Q. 5 Find the Laplace transform of the $erf\sqrt{t}$

Q. 6 Find a function $f(t)$ for each of the given function $F(s)$ given bellow such that $L\{f(t)\}=F(s)$
(i) $\dfrac{1}{\sqrt{s^2+4}}$ **(ii)** $\dfrac{1}{s^{7/2}}$ **(iii)** $\dfrac{5}{s^2}+\left(\dfrac{\sqrt{s}-1}{s}\right)^2-\dfrac{7}{3s+2}$ **(iv)** $\dfrac{3s+7}{s^2-2s-3}$ **(v)** $\dfrac{32}{(16s^2+1)^2}$ **(vi)** $\dfrac{e^{4-3s}}{(s+4)^{5/2}}$ **(vii)** $\dfrac{s^2}{(s^2+4)^2}$

Q. 7 Find the values of the following by using the convolution theorem
(i) $L^{-1}[\dfrac{s}{(s^2+16)^2}]$ **(ii)** $L^{-1}[\dfrac{1}{(s-2)(s^2+1)}]$ **(iii)** $L^{-1}[\dfrac{1}{s(s^2+4)^2}]$ **(iv)** $L^{-1}[\cot^{-1}\dfrac{s+3}{2}]$ **(v)** $L^{-1}[\log\dfrac{s+a}{s+b}]$

Q. 8 Find solution of each of the following initial value problem by using Laplace transform

$(a)\quad y''-3y'+2y=4t+e^{3t} \qquad y(0)=1,\ y'(0)=-1$

$(b)\quad y'''+2y''-y'-2y=0 \qquad y(0)=1,\ y'(0)=2,\ y''(0)=2$

$(c)\quad t\,y''+(1-2t)y'-2y=0 \qquad y(0)=1,\ y'(0)=2$

$(d)\quad y''-ty'+y=1 \qquad y(0)=1,\ y'(0)=2$

$(e)\quad y'+3y=\begin{cases}t, & 0\leq t<1\\ 0, & t\geq 1\end{cases} \qquad y(0)=2$

$(f)\quad y''+5y'+6y=1-u(t-3)-u(t-5) \quad y(0)=0,\ y'(0)=0$

Q. 9 Solve the following simultaneous equations by using Laplace transform

$(a)\quad (D^2-3)x-4y=0,\ x+(D^2+1)y=0$ given that $x=y=\dfrac{dy}{dt}=0$ and $\dfrac{dx}{dt}=2$ at $t=0; D\equiv\dfrac{d}{dt}$

$(b)\quad (D^2-1)x+5Dy=t,\ 2Dx-(D^2-4)y=2$ given that $x=y=\dfrac{dx}{dt}=\dfrac{dy}{dt}=0$ at $t=0;\ D\equiv\dfrac{d}{dt}$

Q. 10 Solve the following initial-boundary value problems.

$(a).\ u_x+x\,u_t=0, \quad u(x,0)=1,\ u(0,t)=t$

$(b).\ u_{tt}=u_{xx},\ x>0,\ t>0,\ u(x,0)=e^{-x},\ u_t(x,0)=0,\ u(0,t)=0,\ u(x,t)\ is\ bounded\ as\ x\to\infty$

Answers:

1. i. $\frac{162}{(s^2-81)(s^2-9)}$, **ii.** $\frac{3(s^2-58)}{(s^2+58)^2-36s^2}$, **iii.** $\frac{10(3s^2+6s+2)}{(s^2+6s+34)^2}$, **iv.** $\frac{30(s+4)}{(s^2+8s+20)(s^2+8s+80)}$.

2. i. $\frac{2}{s}(1+\frac{e^{-s}}{s})$ **ii.** $\frac{e^{-s\pi/3}}{s^2+1}$ **iii.** 0. **3. (i)** $\sqrt{\frac{\pi}{s}}e^{-1/4s}$ **(ii)** $\tan^{-1}(1/s)$ **(iii)** no; **4. (i)** $\frac{\pi}{2}$ **(ii)** 0 **(iii)** (¼) log 5; **5.** $\frac{1}{s\sqrt{s+1}}$

6i. $(1/2)\sin 2t$; **ii.** $(8/15)\sqrt{t/\pi}$; **iii.** $6t+1-4\sqrt{t/\pi}-(7/3)e^{-27/3}$; **iv.** $4e^{3t}-e^{-t}$;

v. $(t/4)\sin(t/4)$; **vi.** $(4/3\sqrt{\pi})(t-3)^{3/2}\,e^{-4(t-4)}\,U(t-3)$; **vii.** $(1/4)(\sin 2t+2t\cos 2t)$;

7i. $\frac{t\sin 4t}{8}$; **ii.** $(1/5)(e^{2t}-2\sin t-\cos t)$; **iii.** $(1/16)(1-t\sin 2t-\cos 2t)$; **(iv)** $\frac{e^{-3t}\sin 2t}{t}$; **(v)** $\frac{e^{-bt}-e^{-at}}{t}$;

8 a. $3+2t+(1/2)(e^{3t}-e^{t})-2e^{2t}$; **b.** $(1/3)(5e^{t}+e^{-2t})-e^{-t}$; **c.** e^{2t};

d. $2t+1$; **e.** $[(3t-1+19e^{-3t})u(t)+(1-3t+2e^{-3(t-1)})u(t-1)]/9$; **f.** $(1/6)[1+2e^{-3t}-3e^{-2t}]u(t)$
$-(1/6)[1+2e^{-3(t-3)}-3e^{-2(t-3)}]u(t-3)-(1/6)[1+2e^{-3(t-5)}-3e^{-2(t-5)}]u(t-5)$;

9. a. $x=2t\cosh t,\ y=(1-t)\sinh t$; **b.** $x=-t+5\sin t-2\sin 2t,\ y=1-2\cos t+\cos 2t$

10. a. $u(x,\ t)=1+[(t-x^2/2)-1]u(t-x^2/2)$; **b.** $u(x,\ t)=e^{-x}\cosh t-\cosh(t-x)u(t-x)$