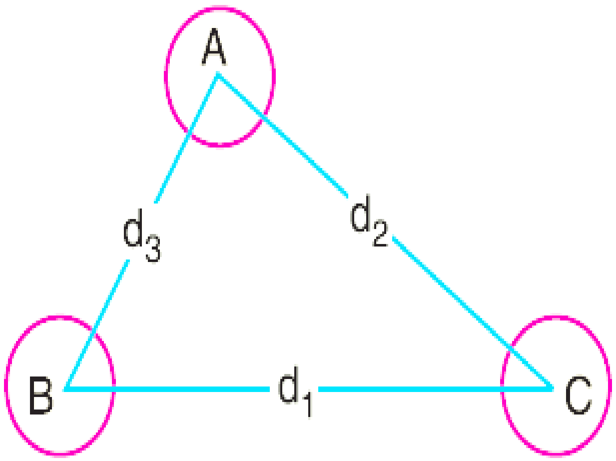
Inductance of a 3-Phase Overhead Line



- The three conductors *A*, *B* and *C* of a 3-phase line carrying currents I_A, I_B and I_C respectively.
- Let *d*1, *d*2 and *d*3 be the spacings between the conductors as shown.
- Let us further assume that the loads are balanced *i.e.* $I_A + I_B + I_C = 0$.
- Consider the flux linkages with conductor *A.*
- There will be flux linkages with conductor *A* due to its own current and also due to the mutual inductance effects of I_B and I_C.

- Flux linkages with conductor *A* due to its own current

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_r^{\infty} \frac{dx}{x}\right) \quad \ldots(i)$$

Inductance of a 3-Phase Overhead Line

- Flux linkages with conductor *A* due to current I_B $= \frac{\mu_0 I_B}{2\pi} \int_{d_3}^{\infty} \frac{dx}{x}$ ...(*ii*)
- Flux linkages with conductor *A* due to current I_C $= \frac{\mu_0 I_C}{2\pi} \int_{d_2}^{\infty} \frac{dx}{x}$ ...(*iii*)
- Total flux linkages with conductor *A* is $\psi_A = (i) + (ii) + (iii)$

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_{r}^{\infty} \frac{dx}{x}\right) + \frac{\mu_0 I_B}{2\pi} \int_{d_3}^{\infty} \frac{dx}{x} + \frac{\mu_0 I_C}{2\pi} \int_{d_2}^{\infty} \frac{dx}{x}$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} + \int_{r}^{\infty} \frac{dx}{x}\right) I_A + I_B \int_{d_3}^{\infty} \frac{dx}{x} + I_C \int_{d_2}^{\infty} \frac{dx}{x}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d_3 - I_C \log_e d_2 + \log_e \infty \left(I_A + I_B + I_C\right)\right]$$

As $I_A + I_B + I_C = 0$, $\therefore$ $\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d_3 - I_C \log_e d_2\right]$

Inductance of a 3-Phase Overhead Line

(*i*) Symmetrical spacing

- If the three conductors *A*, *B* and *C* are placed symmetrically at the corners of an equilateral triangle of side *d*, then, $d_1 = d_2 = d_3 = d$.

- Under such conditions, the flux linkages with conductor *A* become :

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d - I_C \log_e d\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - \left(I_B + I_C\right) \log_e d\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A + I_A \log_e d\right] \qquad (\because I_B + I_C = -I_A)$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{ werber-turns/m}$$

Inductance of conductor *A*, $L_A = \frac{\psi_A}{I_A} \text{ H / m} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{ H/m}$

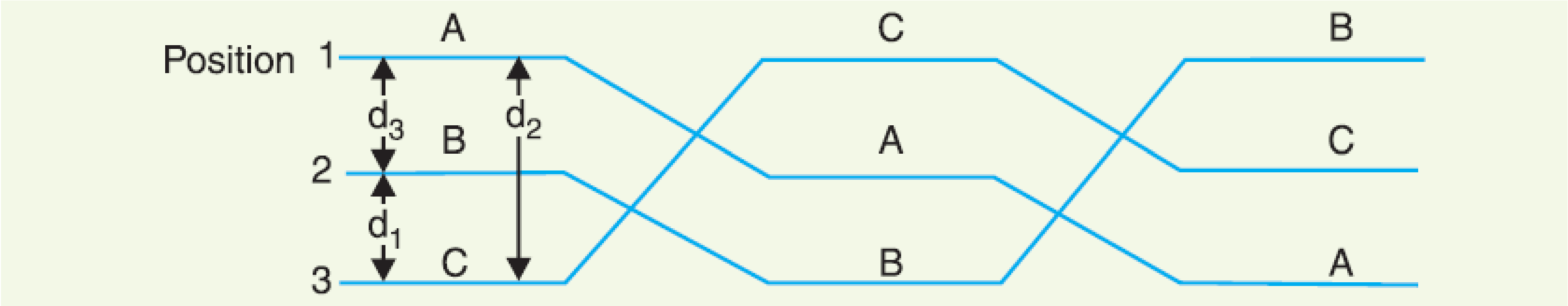
$$= \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{ H/m} \qquad L_A = 10^{-7}\left[0\cdot 5 + 2\log_e \frac{d}{r}\right] \text{ H/m}$$

- Derived in a similar way, the expressions for inductance are the same for conductors *B* and *C*.

Inductance of a 3-Phase Overhead Line

(*ii*) Unsymmetrical spacing

- When 3-phase line conductors are not equidistant from each other, the conductor spacing is said to be unsymmetrical.
- Under such conditions, the flux linkages and inductance of each phase are not the same.
- A different inductance in each phase results in unequal voltage drops in the three phases even if the currents in the conductors are balanced.
- Therefore, the voltage at the receiving end will not be the same for all phases.
- In order that voltage drops are equal in all conductors, we generally interchange the positions of the conductors at regular intervals along the line so that each conductor occupies the original position of every other conductor over an equal distance.
- Such an exchange of positions is known as *transposition.*



- The phase conductors are designated as *A*, *B* and *C* and the positions occupied are numbered 1, 2 and 3.
- The effect of transposition is that each conductor has the same average inductance.

Inductance of a 3-Phase Overhead Line

- Above Fig. shows a 3-phase transposed line having unsymmetrical spacing.
- Let us assume that each of the three sections is 1 m in length. Let us further assume balanced conditions *i.e.*,

$$I_A + I_B + I_C = 0.$$

- Let the line currents be :

$$I_A = I\,(1 + j\,0)$$
$$I_B = I\,(-\,0{\cdot}5 - j\,0{\cdot}866)$$
$$I_C = I\,(-0{\cdot}5 + j\,0{\cdot}866)$$

- As proved above, the total flux linkages per metre length of conductor *A* is

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d_3 - I_C \log_e d_2\right]$$

- Putting the values of I_A, I_B and I_C, we get,

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I - I(-0{\cdot}5 - j\,0{\cdot}866)\log_e d_3 - I(-0{\cdot}5 + j\,0{\cdot}866)\log_e d_2\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} I - I \log_e r + 0{\cdot}5\,I \log_e d_3 + j\,0{\cdot}866 \log_e d_3 + 0{\cdot}5\,I \log_e d_2 - j\,0{\cdot}866\,I \log_e d_2\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} I - I \log_e r + 0{\cdot}5\,I\left(\log_e d_3 + \log_e d_2\right) + j\,0{\cdot}866\,I\left(\log_e d_3 - \log_e d_2\right)\right]$$

Inductance of a 3-Phase Overhead Line

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4}I - I\log_e r + I^{*}\log_e\sqrt{d_2 d_3} + j\,0\cdot 866\,I\log_e\frac{d_3}{d_2}\right]$$

---

* $0\cdot 5\,I\,(log_e d_3 + \log_e d_2) = 0\cdot 5\,I\log_e d_2 d_3 = I\log_e (d_2 d_3)^{0\cdot 5} = I\,log_e\sqrt{d_2 d_3}$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4}I + I\log_e\frac{\sqrt{d_2\,d_3}}{r} + j\,0\cdot 866\,I\log_e\frac{d_3}{d_2}\right]$$

$$= \frac{\mu_0\,I}{2\pi}\left[\frac{1}{4} + \log_e\frac{\sqrt{d_2 d_3}}{r} + j\,0\cdot 866\log_e\frac{d_3}{d_2}\right]$$

$\therefore$ Inductance of conductor A is

$$L_A = \frac{\psi_A}{I_A} = \frac{\psi_A}{I} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e\frac{\sqrt{d_2\,d_3}}{r} + j\,0\cdot 866\log_e\frac{d_3}{d_2}\right]$$

$$= \frac{4\pi\times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e\frac{\sqrt{d_2\,d_3}}{r} + j\,0\cdot 866\log_e\frac{d_3}{d_2}\right]\ \text{H/m}$$

$$= 10^{-7}\left[\frac{1}{2} + 2\log_e\frac{\sqrt{d_2\,d_3}}{r} + j\,1\cdot 732\log_e\frac{d_3}{d_2}\right]\ \text{H/m}$$

Inductance of a 3-Phase Overhead Line

- Similarly inductance of conductors B and C will be :

$$L_B = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt{d_3 d_1}}{r} + j\,1\cdot732\log_e \frac{d_1}{d_3}\right] \text{H/m}$$

$$L_C = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt{d_1 d_2}}{r} + j\,1\cdot732\log_e \frac{d_2}{d_1}\right] \text{H/m}$$

- Inducance of each line conductor $= \frac{1}{3}(L_A + L_B + L_C)$

$$= {}^{*}\left[\frac{1}{2} + 2\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}\right] \times 10^{-7} \text{ H/m}$$

$$= \left[0\cdot5 + 2\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}\right] \times 10^{-7} \text{ H/m}$$

- If we compare the formula of inductance of an unsymmetrically spaced transposed line with that of symmetrically spaced line, we find that inductance of each line conductor in the two cases will be equal if $d = \sqrt[3]{d_1 d_2 d_3}$
- The distance d is known as *equivalent equilateral spacing* for unsymmetrically transposed line.