Chapter
4

Numerical Differentiation

So far, we were finding the polynomial curve y = f(x) passing through the ordered pairs (x_0, y_0), (x_1, y_1)............. (x_n, y_n).Now we find the derivative value of such polynomials.
To get the derivatives we first find out y = f(x) through the points and then differentiate.

If the arguments are equally spaced, we apply Newton's difference formula.

If the derivative is required at the point nearer to the beginning value, we use Newton's forward difference formula. If we require the derivatives at the end of the table we use Newton's backward interpolation formula. If the value of the derivative is required near the middle of the table, we use stirling interpolation formula.

In case of unequal intervals, we use Newtons divided difference formula or Lagranges Interpolation formula.

Newton's forward difference formula to get the derivative:
By Newton's forward difference interpolation formula

$$f(x) = f(a+ph) = f(a) + p\Delta f(a) + \frac{p(p-1)}{2!}\Delta^2 f(a)$$

$$+\frac{p(p-1)(p-2)}{3!}\Delta^3 f(a) + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 f(a) + \text{............}(1)$$

where $p = \frac{x-a}{h}$.

Differentiating (1) with respect to *p*, we get

$$f'(x) = f'(a+ph)(\mathrm{h}) = \Delta f(a) + \frac{2p-1}{2!}\Delta^2 f(a)$$

$$+\frac{3p^2-6\mathrm{p}+2}{3!}\Delta^3 f(a) + \frac{4p^3-18\mathrm{p}^2+22\mathrm{p}-6}{4!}\Delta^4 f(a) + \ldots\ldots\ldots\ldots(2)$$

$$\Rightarrow f'(x) = \frac{1}{h}\left[\Delta f(a) + \frac{2p-1}{2}\Delta^2 f(a) + \frac{3p^2-6p+2}{6}\Delta^3 f(a) + \ldots\ldots\ldots\ldots\right]$$

If x = a, then x = a + ph $\Rightarrow$ p = 0

Putting p = 0 in (2), we get

$$f'(x) = \frac{1}{h}\left[\Delta f(a) - \frac{\Delta^2}{2} f(a) + \frac{1}{3}\Delta^3 f(a) - \frac{1}{4}\Delta^4 f(a) + \ldots\ldots\ldots\ldots\right]$$

Again differentiating with respect to *p*, we get

$$f'(a+ph)\times h = \frac{1}{h}\left[\begin{array}{l}\Delta^2 f(a) + (p-1)\Delta^3 f(a) \\ +\dfrac{12p^2-36p+22}{4!}\Delta^4 f(a) + \ldots\ldots\ldots\end{array}\right]$$

Or

$$f''(\mathrm{x}) = f''(a+ph) = \frac{1}{h^2}\left[\begin{array}{l}\Delta^2 f(a) + (p-1)\Delta^3 f(a) + \dfrac{6p^2-18p+11}{12}\Delta^4 f(a) \\ +\dfrac{2p^3-12p^2+21p-10}{12}\Delta^5 f(a) + \ldots\ldots\ldots\end{array}\right] \quad ..(3)$$

Problem: Given that

x	1.0	1.1	1.2	1.3
y	0.841	0.891	0.932	0.963

Find $\frac{dy}{dx}$ at x=1.0

Solution: Newton's forward difference table

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1.0	0.841			
1.1	0.891	0.050		
1.2	0.932	0.041	-0.009	-0.001
1.3	0.963	0.031	-0.010	

$x = 1, a = 1, h = 0.1, x = a + ph \Rightarrow 1 = 1 + p \times 0.1 \Rightarrow p = 0$

By Newton's forward difference interpolation formula.

$$f(x) = f(a+ph) = f(a) + p\Delta f(\text{a}) + \frac{p(p-1)}{2!}\Delta^2 f(a) + \frac{p(p-1)(\text{p}-2)}{3!}\Delta^3 f(a) + ...$$

$$f'(a+ph) = \frac{1}{h}\left[\Delta f(\text{a}) + \frac{2p-1}{2}\Delta^2 f(a) + \frac{3p^2 - 6\text{p} + 2}{6}\Delta^3 f(a)\right]$$

Putting value of $p = 0$

$$f'(\text{x}) = \frac{1}{h}\left[\Delta f(\text{a}) - \frac{1}{2}\Delta^2 f(a) + \frac{1}{3}\Delta^3 f(a)\right]$$

$$f'(1) = \frac{1}{0.1}\left[\Delta f(1) - \frac{1}{2}\Delta^2 f(1) + \frac{1}{3}\Delta^3 f(1)\right]$$

$$= 10\left[0.05 - 0.5(-0.009) + \frac{1}{3}(-0.001)\right] \quad [h=0.1]$$

$$= 10\left[0.05 + 0.0045 - 0.0003\right]$$

$$= 10\left[0.0545 - 0.0003\right] = 10 \times (0.0542)$$

$\frac{dy}{dx} = 0.542$ at x=1.

Problem: From the following table, obtain the value of $\frac{d^2y}{dx^2}$ at the point x = 0.96,

x	0.96	0.98	1.00	1.02	1.04
y	0.7825	0.7739	0.7651	0.7563	0.7473

Solution: Newton's Forward difference table.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.96	0.7825				
0.98	0.7739	- 0.0086	- 0.0002		
1.00	0.7651	- 0.00088	- 0.0000	0.0002	- 0.0004
1.02	0.7563	- 0.0088	- 0.0002	- 0.0002	
1.04	0.7473	- 0.0090			

a = 0.96, h = 0.02, a + ph = 0.96
$0.96 + p \times 0.02 = 0.96 \quad \Rightarrow p = 0$

$$f''(a+ph) = \frac{1}{h^2}\left[\Delta^2 f(a) + (p-1)\Delta^3 f(a) + \frac{1}{24}(12p^2 - 36p + 22)\Delta^4 f(a)\right]$$

$$= \frac{1}{(0.02)^2}\left[-0.0002 + (0-1)(0.0002) + \frac{1}{24}(0-0+22)(-0.0004)\right]$$

$$= 2500[-0.0002 - 0.0002 - 0.00037]$$

$$= 2500 \times 0.00077 = -1.925$$

Hence $\frac{d^2y}{dx^2} = -1.925$ at the point x = 0.96.

Problem: Find the 1st and 2nd derivative of the function given below at the point x=1.2

x	1	2	3	4	5
y	0	1	5	6	8

Solution: Newton's forward difference table

x	**f(x)**	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1	0	1			
2	1	4	3	-6	
3	5	1	-3	4	10
4	6	2	1		
5	8				

$x_p = 1.2, \quad a = 1, \quad h = 1, \quad x_p = a + ph \Rightarrow 1.2 = 1 + p \Rightarrow p = 0.2$

By Newton's forward difference interpolation formula.

$$f'(x) = f'(a+ph) = \frac{1}{h}\left[\begin{array}{l}\Delta f(a) + \frac{2p-1}{2}\Delta^2 f(a) + \frac{3p^2-6p+2}{6}\Delta^3 f(a) \\ + \frac{4p^3-18p^2+22p-6}{24}\Delta^4 f(a)\end{array}\right]$$

$$f'(1.2) = \frac{1}{1}\left[\begin{array}{l}1 + \frac{2\times 0.2-1}{2}\times 3 + \frac{3(0.2)^2-6(0.2)+2}{6}(-6) \\ + \frac{4(0.2)^3-18(0.2)^2+22(0.2)-6}{24}\times 10\end{array}\right]$$

$$= \left[1 - \frac{0.6\times 3}{2} - (0.12-1.2+2) + (0.032-0.72+4.4-6)\frac{5}{12}\right]$$

$$= \left[1-0.9-0.92-0.9533\right] = -1.7733$$

Hence Ist derivative of the function f(x) is -1.7733 at x=1.2

Second derivative at $x = 1.2$

$$f''(x) = \frac{1}{h^2}\left[\Delta^2 f(a) + (p-1)\Delta^3 f(a) + \frac{6p^2-18p+11)}{12}\Delta^4 f(a)\right]$$

$$= \frac{1}{1^2}\left[3 + (0.2-1)(-6) + \frac{6\times(0.2)^2-18(0.2)+11)}{12}\times 10\right]$$

$$= 3 + 4.8 + \frac{0.24-3.6+11)}{12}\times 10$$

$$= 3 + 4.8 + 6.3667 = 14.1667$$

Hence the second derivative of the function *f(x)* is 14.667 at $x =$ 1.2

Newton's Backward Differentiation.

Newton's backward differentiation formula is

$$f(x) = f(x_n + ph) = f(x_n) + p\nabla f(x_n) + \frac{p(p+1)}{2!}\nabla^2 f(x_n) + \ldots\ldots\ldots\ldots$$

Differentiating with respect to p, we get

$$hf'(x_n + ph) = \nabla f(x_n) + \frac{2p+1}{2!}\nabla^2 f(x_n) + \frac{3p^2+6p+2}{3!}\nabla^3 f(x_n)$$

$$+ \frac{4p^3+18p^2+22p+6}{4!}\nabla^4 f(x_n) + \ldots\ldots\ldots\ldots$$

So, $$f'(x) = f'(x_n + ph) = \frac{1}{h}\left[\begin{array}{l} \nabla f(x_n) + \frac{2p+1}{2!}\nabla^2 f(x_n) + \frac{3p^2+6p+2}{6}\nabla^3 f(x_n) \\ + \frac{2p^3+9p^2+11p+3}{12}\nabla^4 f(x_n) + \ldots\ldots\ldots\ldots \end{array}\right]$$

Again differentiating with respect to p, we get

$$f''(x_n + ph) = \frac{1}{h^2}\left[\begin{array}{l} \nabla^2 f(x_n) + (\mathrm{p}+1)\nabla^3 f(x_n) \\ + \frac{6p^2+18p+2}{12}\nabla^4 f(x_n) + \ldots\ldots\ldots\ldots \end{array}\right]$$

Problem: Given that

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.481	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at (1) x = 1.1 and (2) 1.6

Solution: The difference table is as under.

x	**y**	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$	$\nabla^6 y$
1.0	7.989						
1.1	8.403	0.414	-0.036	0.006			
1.2	8.781	0.378	-0.030	0.004	-0.002		
1.3	9.129	0.348	-0.026	0.003	-0.001	0.001	0.002
1.4	9.451	0.322	-0.023	0.005	0.002	0.003	
1.5	9.750	0.299	-0.018				
1.6	10.031	0.281					

a = 1.1, x=1.1, h = 0.1x= a + ph $\Rightarrow$ 1.1=1.1+ p×0.1$\Rightarrow$p = 0

$$\frac{dy}{dx}=\frac{1}{h}\left[\Delta y+\frac{1}{2!}(2p-1)\Delta^2 y+\frac{1}{3!}(3p^2-6p+2)\Delta^3 y+\right.$$
$$+\frac{4p^3-18p^2+22p-6}{4!}\Delta^5 y+..........$$
$$+\frac{6p^5-75p^4+340p^3-675p^2+548p-120}{6!}\Delta^6 y+..........(1)$$

On Putting the values of p = 0 in (1), we get,

$$\frac{dy}{dx}=\frac{1}{h}\left[\Delta y+\frac{1}{2}\Delta^2 y+\frac{1}{3}\Delta^3 y-\frac{1}{4}\Delta^4 y+\frac{1}{5}\Delta^5 y-\frac{1}{6}\Delta^6 y+.......\right]$$

So,

$$\left(\frac{dy}{dx}\right)_{1.1}=\frac{1}{0.1}\left[0.378-\frac{1}{2}(-0.03)+\frac{1}{3}(0.004)-\frac{1}{4}(-0.001)+\frac{1}{5}(0.003)+.......\right]$$
$$=\frac{1}{0.1}\left[0.378+0.015+0.0013+0.0003+0.0006\right]$$
$$=\frac{1}{0.1}\left[0.3952\right]=3.952$$

$$\frac{d^2y}{dx^2}=\frac{1}{h^2}\left[\begin{array}{l}\Delta^2 y+(p-1)\Delta^3 y+\dfrac{6p^2-18p+11}{12}\Delta^4 y+\dfrac{2p^3-12p^2+21p-10}{12}\Delta^5 y\\+\dfrac{15p^4-150p^3+510p^2-675p+274}{360}\Delta^6 y+........\end{array}\right]$$

When $p = 0$

$$\frac{d^2y}{dx^2}=\frac{1}{h^2}\left[\Delta^2 y-\Delta^3 y+\frac{11}{12}\Delta^4 y-\frac{5}{6}\Delta^5 y+\frac{137}{180}\Delta^6 y+.......\right] \qquad (2)$$

Putting the values in (2) we get.

$$\left(\frac{d^2y}{dx^2}\right)_{1.1}=\frac{1}{(0.1)^2}\left[-0.030-0.004+\frac{11}{12}(-0.001)-\frac{5}{6}(0.003)\right]$$

$$=\frac{1}{0.01}\left[-0.30-0.004-0.0009-0.0025\right]$$
$$=\frac{1}{0.01}\left[-0.0374\right]=-3.74$$

Stirling Formula for derivatives:

$$f(a+ph)=f(a)+p\left[\frac{\Delta f(a)+\Delta f(a-h)}{2}\right]+\frac{p^2}{2!}\Delta^2 f(a-h)$$

$$+\ \frac{p(p^2-1)}{3!}\left[\frac{\Delta^3 f(a-h)+\Delta^3 f(a-2h)}{2}\right]$$

$$+\ \frac{p^2(p^2-1)}{4!}\Delta^4 f(a-2h)+\ldots\ldots\ldots\ldots(1)$$

$$hf'(a+ph)=\left[\frac{\Delta f(a)+\Delta f(a-h)}{2}\right]+p\Delta^2 f(a-h)$$

$$+\ \frac{3p^2-1}{6}\left[\frac{\Delta^3 f(a-h)+\Delta^3 f(a-2h)}{2}\right]$$

$$+\ \frac{4p^3-2p}{24}\Delta^4 f(a-2h)+\ldots\ldots\ldots\ldots$$

$$\Rightarrow f'(a+ph)=\frac{1}{h}\left[\left\{\frac{\Delta f(a)+\Delta f(a-h)}{2}+p\Delta^2 f(a-h)\right\}\right]$$

$$+\ \frac{3p^2-1}{6}\left[\frac{\Delta^3 f(a-h)+\Delta^3 f(a-2h)}{2}\right]+\frac{2p^3-p}{12}\Delta^4 f(a-2h)$$

$$\Rightarrow f''(a+ph)=\frac{1}{h^2}\left[\Delta^2 f(a-h)+\left\{\frac{\Delta^3 f(a-h)+\Delta^3 f(a-2h)}{2}\right\}\right]$$

$$+\ \frac{6p^2-1}{12}\left\{\Delta^4 f(a-2h)\right\}+\ldots\ldots\ldots\ldots$$

Problem: A slider in a machine moves along a fixed straight rod. Its distance x cm. along the rod is given below for various values of the time t seconds. Find the velocity of the slider and its acceleration when t = 0.3 sec.

t	0	0.1	0.2	0.3	0.4	0.5	0.6
x	30.13	31.62	32.87	33.64	33.95	33.81	33.24

Solution: We have to determine velocity $\left(\frac{dx}{dt}\right)$ and acceleration $\left(\frac{d^2x}{dt^2}\right)$ at the point t = 0.3 sec., which is near the middle of the table, therefore central difference formula, will be used.

***D*ifference table is as follows:**

p	t	x	∇	∇^2	∇^3	∇^4	∇^5	∇^6
-3	0	30.13						
-2	0.1	31.62	1.42	-0.24	-0.24			
-1	0.2	32.87	1.25	-0.48	0.02	0.26		
0	0.3	33.64	0.77	-0.46	0.01	-0.01	0.027	0.29
1	0.4	33.95	0.31	-0.45	0.02	0.02	0.02	
2	0.5	33.81	-0.14	-0.43				
3	0.6	33.24	-0.57					

$$a = 0.3, \quad a + ph = 0.3 \Rightarrow 0.3 + p(0.1) = 0.3 \Rightarrow p = 0$$

By Stirling formula.

$$f'(a+ph) = \frac{1}{h}\left[\frac{\Delta f(a) + \Delta f(a-h)}{2}\right] + p\Delta^2 f(a-h)$$

$$+ \frac{3p^2-1}{6}\left[\frac{\Delta^3 f(a-h) + \Delta^3 f(a-2h)}{2}\right] + \frac{2p^3-p}{12}\Delta^4 f(a-2h) + ...(1)$$

Putting the value of p = 0 in (1), we get

$$f'(a) = \frac{1}{h}\left[\frac{\Delta f(a) + \Delta f(a-h)}{2} - \frac{1}{6}\left(\frac{\Delta^3 f(a-h) + \Delta^3 f(a-2h)}{2}\right)\right]$$

$$+ \frac{1}{30}\left[\frac{\Delta^5 f(a-2h) + \Delta^5 f(a-3h)}{2}\right] + ...$$

So

$$f'(0.3) = \frac{1}{0.1}\left[\frac{0.31+0.77}{2} - \frac{1}{6}\left(\frac{0.01+0.02}{2}\right) + \frac{1}{30}\left(\frac{0.02-0.27}{2}\right)\right]$$

$$f'(0.3) = \frac{1}{0.1}\left[0.54 - 0.0025 - 0.004167\right]$$
$$= 5.333 cm/\sec.$$

Differentiating (1) with respect to p, we get

$$f''(a+ph) = \frac{1}{h^2}\begin{bmatrix} \Delta^2 f(a-h) + \mathrm{p}\dfrac{\Delta^3 f(a-h) + \Delta^3 f(a-2h)}{2} \\ + \dfrac{6p^2-1}{12}\Delta^4 f(a-2h) + \ldots\ldots.. \end{bmatrix}$$

Putting $p = 0$, we get

$$f''(a) = \frac{1}{h^2}\left[\Delta^2 f(a-h) - \frac{1}{12}\Delta^4 f(a-2h) + \frac{1}{90}\Delta^6 f(a-3h)\right]$$

$$f''(0.3) = \frac{1}{(0.1)^2}\left[-0.46 - \frac{1}{12}(-0.01) + \frac{1}{90}(0.29)\right]$$
$$=100\left[-0.46 + 0.00083 + 0.00322\right]$$
$$=100\left[-0.45595\right]$$
$$=-45.595$$
$$=-45.6 \text{ cm/sec}^2$$

Newton's Divided Difference formula for derivatives:

Newton's divided difference formula is

$$f(x) = f(x_0) + (x - x_0)\Delta f(x_0) + (x - x_0)(x - x_1)\Delta^2 f(x_0)$$
$$+ (x - x_0)(x - x_1)(x - x_2)\Delta^2 f(x_0) + (x - x_0)(x - x_1)(x - x_2)$$
$$+ (x - x_3)\Delta^4 f(x_0) \text{ upto third differences.}$$

On differentiating with respect to *x*, we get

$$f'(x) = \Delta f(x_0) + [2x - x_0 - x_1]\Delta^2 f(x_0)$$
$$+ [3x^2 - 2x(x_0 + x_1 + x_2) + x_0 x_1 + x_0 x_2 + x_1 x_2]\Delta^3 f(x_0)$$
$$+ [4x^3 - 3x^2(x_0 + x_1 + x_2 + x_3) + 2x(x_0 x_1 + x_0 x_2 + x_0 x_3$$
$$+ x_1 x_2 + x_1 x_3 + x_2 x_3) + x_0 x_1 x_2 + x_0 x_1 x_3 + x_0 x_2 x_3$$
$$+ x_1 x_2 x_3]\Delta^4 f(x_0)$$

$$f''(x) = 2\Delta^2 f(x_0) + [6x - 2(x_0 + x_1 + x_2)]\Delta^2 f(x_0)$$
$$+ [12x^2 - 6x(x_0 + x_1 + x_2 + x_3)$$
$$+ 2(x_0 x_1 + x_0 x_2 + x_0 x_3 + x_1 x_2 + x_1 x_3 + x_2 x_3)]\Delta^4 f(x_0)$$

$$f'''(x) = 6\Delta^3 f(x_0) + [24x - 6(x_0 + x_1 + x_2 + x_3)]\Delta^4 f(x_0)$$

Problem: From the following table find the Ist derivative at x = 4 using Newton's divided difference formula.

x	1	2	4	8	10
f(x)	0	1	5	21	27

Solution: The divided difference table is as follows:

x	***f(x)***	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1	0	$\frac{1-0}{2-1}=1$	$\frac{2-1}{4-1}=\frac{1}{3}$	$\frac{\frac{1}{3}-\frac{1}{3}}{8-1}=0$	$\frac{-\frac{1}{16}-0}{10-1}=-\frac{1}{144}$
2	1	$\frac{5-1}{4-2}=2$	$\frac{4-2}{8-2}=\frac{1}{3}$	$\frac{-\frac{1}{6}-\frac{1}{3}}{10-2}=-\frac{1}{16}$	
4	5	$\frac{21-5}{8-4}=4$	$\frac{3-4}{10-4}=-\frac{1}{6}$		
8	21	$\frac{27-21}{10-8}=3$			
10	27				

Newton's divided difference formula is:

$$f(x)=f(x_0)+(x-x_0)\Delta f(x_0)+(x-x_0)(x-x_1)\Delta^2 f(x_0)$$
$$+\ (x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x_0)+(x-x_0)(x-x_1)(x-x_2)$$
$$+\ (x-x_3)\Delta^4 f(x_0)+\text{........................}(1)$$

$$f'(x)=\Delta f(x_0)+[2x-x_0-x_1]\Delta^2 f(x_0)+[(x-x_1)(x-x_2)$$
$$+(x-x_0)(x-x_2)+(x-x_0)(x-x_1)]\Delta^3 f(x_0)$$
$$+[(x-x_1)(x-x_2)+(x-x_0)(x-x_2)(x-x_3)$$
$$+(x-x_0)(x-x_1)(x-x_3)+(x-x_0)(x-x_1)(x-x_2)]\Delta^4 f(x_0)\text{............}(2)$$

Here $x=4,\ x_0=1,\ x_1=2,\ x_2=4,\ x_3=8,\ x_4=10$

$$\Delta f(x_0)=1,\ \Delta^2 f(x_0)=\frac{1}{3},\ \Delta^3 f(x_0)=0,\ \Delta^4 f(x_0)=-\frac{1}{1.44}$$

Putting these values in (2) we get,

$$f'(4) = 1 + [8 - 1 = 2]\frac{1}{3} + [(4-2)(4-4) + (4-1)(4-1)(4-2)$$
$$+ (4-1)(4-2)] \times 0 + [(4-2)(4-4)(4-8) + (4-1)(4-4)(4-8)$$
$$+ (4-1)(4-2)(4-8) + (4-1)(4-2)(4-4)](-\frac{1}{144})$$
$$= 1 + \frac{5}{3} + 0 + \left[2\times 0\times(-4) + 3\times 0\times(-4) + 3\times 2\times(-4) + 3\times 2\times 0\right]\left(-\frac{1}{144}\right)$$
$$= 1 + \frac{5}{3} + \frac{1}{6} = \frac{17}{6} = 2.833$$

Lagrange's Method for derivatives:

Problem: Find out first derivative and second derivative from the following data by Lagrange's method.

x	1	3	-4
f(x)	3	-5	4

Solution: Lagrange's formula for interpolation is

$$f(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} f(x_1) + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} f(x_2)$$
$$+ \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)} f(x_3)$$

Here $x_1=1$ $x_2=2$ $x_3=-4$

$f(x_1)=3$ $f(x_2)=-5$ $f(x_3)=4$

$$\Rightarrow f(x) = \frac{(x-2)(x+4)}{(1-2)(1+4)} \times 3 + \frac{(x-1)(x+4)}{(2-1)(2+4)}(-5)$$
$$+ \frac{(x-1)(x-2)}{(-4-1)(-4-2)} \times 4$$

$$= (x^2+2x-8)\left(-\frac{3}{5}\right) + (x^2+3x-4)\left(-\frac{5}{6}\right) + (x^2-3x+2)\left(\frac{4}{30}\right)$$

$$= \left(-\frac{3}{5}-\frac{5}{6}+\frac{2}{15}\right)x^2 + \left(-\frac{6}{5}-\frac{15}{6}-\frac{4}{10}\right)x + \left(\frac{24}{5}+\frac{10}{3}+\frac{7}{15}\right)$$

$$\Rightarrow f(x) = -\frac{13}{10}x^2 - \frac{41}{10}x + \frac{42}{5} = -\frac{1}{10}(13x^2+41x-84)$$

Differentiating with respect to x, we get

$$f'(x) = -\frac{1}{10}(26x+41)$$

Again with respect to x, $f''(x) = -\frac{1}{10}\times 26 = -\frac{13}{5}$

Exercise

1. Given the following data, find $y'(6)$ and the maximum value of y (if it exists)

x:	0	2	3	4	7	9
y:	4	26	58	112	466	922

2. Find first and second derivative of the function tabulated below at x = 0.6

x:	0.4	0.5	0.6	0.7	0.8
y:	1.5836	1.7974	2.0442	2.3275	2.6511

3. Compute $f'(3)$ from the following table:

x:	1	2	4	8	10
y:	0	1	5	21	27

4. A rocket is launced from the ground, find it's velocity and acceleration, the distance at different time is given below:

t (sec) :	0	1	2	4
x (km) :	0	20	60	120

5. Find the values of d^2y / dx^2 at x = 5 and d^2y / dx^2 at x = 0.5 from the following table:

X:	0	1	2	3	4	5
Y:	4930	5026	5122	5217	5312	5407

Exercise

6. Given the following data find f’ (6)

x	0	2	3	4	7	9
y	4	26	58	112	466	922

7. Find the first and second derivative of the function tabulated below at the point x = 1.5

x	1.5	2.0	2.5	3.0	3.5	4.0
y	3.375	7.0	13.625	24.0	38.875	59.0

8. The table below gives the velocity of a body during the specified time t. Find the acceleration at t =1.1:

t	1.0	1.1	1.2	1.3	1.4
v	43.1	47.7	52.1	56.4	60.8

9. Find $\frac{dy}{dx}$ at x = 1.72 and x=1.76 from the following table:

x	1.72	1.73	1.74	1.75	1.76
y	0.17907	0.17728	0.17552	0.17377	0.17204

10. Find f’ (0) and $\int_0^9 f(x)\,dx$ from the following table:

x	0	2	3	4	7	9
f(x)	4	26	58	110	460	920