Lecture -6
(Initial Value Problem)

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- **Declaration :** — The study material provided in this lecture is original.

Chapter 6

Initial Value problem

Picard's Method:

Consider the differential equation $\frac{dy}{dx} = f(x, y)$ with initial condition $y(x_0) = y_0$ .Integrating with respect to x we get

$$\int_{x_0}^{x_1} \frac{dy}{dx} dx = \int_{x_0}^{x_1} f(x, y) dx$$

Or $y(x_1) - y(x_0) = \int_{x_0}^{x_1} dy = \int_{x_0}^{x} f(t, y(t)) dt$

Or $y(x) = y(x_0) + \int_{x_0}^{x} f(t, y(t)) dt$

Problem: Solve the differential equation by Picard's method $\frac{dy}{dx} = y, \quad y(0) = 1$

Solution: Here $f(x, y) = y$

$$\varphi_1(x) = 1 + \int_0^x \varphi_0(t) dt = 1 + \int_0^x 1 dt = 1 + x$$

$$\varphi_2(x) = 1 + \int_0^x \varphi_1(t) dt = 1 + \int_0^x 1 + t dt = 1 + x + \frac{x^2}{2}$$

$$\varphi_3(x) = 1 + \int_0^x \varphi_2(t) dt = 1 + \int_0^x 1 + t + \frac{t^2}{2} dt = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$$

Nth term is $\varphi_n(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ... + \frac{x^n}{n!}$

Euler's method

In order to use Euler's Method to generate a numerical solution to an initial value problem of the form: $y' = f(x, y)$ with $y(x_o) = y_o$

We divide this interval into small subdivisions of length h. Then, using the initial condition as our starting point, we generate the rest of the solution by using the iterative formulas:

$$x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + h\, f(x_n, y_n)$$

Problem: Solve the initial value problem: $y' = x + 2y$ *with* $y(0) = 0$, finding a value for the solution at $x = 1$ and using steps of size $h = 0.25$.

Solution: The differential equation given tells us the formula for $f(x, y)$ required by the Euler Method, namely: $f(x, y) = x + 2y$

and the initial condition tells us the values of the coordinates of our starting point: $x_o = 0$, $y_o = 0$.

We now use the Euler method formulas to generate values for x_1 and y_1. $x_1 = x_o + h$

Or: $x_1 = 0 + 0.25$ So: $x_1 = 0.25$

And the y-iteration formula, with $n = 0$ gives us:

$y_1 = y_o + h\, f\,(x_o, y_o)$

Or: $y_1 = y_o + h\,(x_o + 2y_o)$

Or: $y_1 = 0 + 0.25\,(0 + 2\text{x}0)$ so: $y_1 = 0$

Summarizing, the second point in our numerical solution is:

- $x_1 = 0.25$
- $y_1 = 0$

We now move on to get the next point in the solution, (x_2, y_2).

The x-iteration formula, with n=1 gives us: $x_2 = x_1 + h$

Or: $x_2 = 0.25 + 0.25$ So: $x_2 = 0.5$

And the y-iteration formula, with $n = 1$ gives us:
$y_2 = y_1 + h\,f(x_1, y_1)$

Or: $y_2 = y_1 + h\,(x_1 + 2y_1)$

Or: $y_2 = 0 + 0.25\,(0.25 + 2\text{x}0)$ so: $y_2 = 0.0625$

Summarizing, the third point in our numerical solution is:

- $x_2 = 0.5$
- $y_2 = 0.0625$

We now move on to get the fourth point in the solution, (x_3, y_3).

The x-iteration formula, with $n = 2$ give us: $x_3 = x_2 + h$

Or: $x_3 = 0.5 + 0.25$ so: $x_3 = 0.75$

And the y-iteration formula, with $n = 2$ give us:
$y_3 = y_2 + h\,f(x_2, y_2)$

Or: $y_3 = y_2 + h\,(x_2 + 2y_2)$

or: $y_3 = 0.0625 + 0.25\,(0.5 + 2\text{x}0.0625)$ so:$y_3 = 0.21875$

Summarizing, the fourth point in our numerical solution is:

- $x_3 = 0.75$
- $y_3 = 0.21875$

We now move on to get the fifth point in the solution, (x_4, y_4).

The x-iteration formula, with $n = 3$ give us: $x_4 = x_3 + h$

Or: $x_4 = 0.75 + 0.25$ so: $x_4 = 1$

And the y-iteration formula, with $n = 3$ give us:
$y_4 = y_3 + h\, f(x_3, y_3)$

Or: $y_4 = y_3 + h\,(x_3 + 2y_3)$

Or: $y_4 = 0.21875 + 0.25\,(0.75 + 2\text{x}0.21875)$ so: $y_4 = 0.515625$

Summarizing, the fourth point in our numerical solution is:

- $x_4 = 1$ $y_4 = 0.515625$

We could summarize the **results** of all of our calculations in a tabular form, as follows:

n	x_n	y_n
0	0.00	0.000000
1	0.25	0.000000
2	0.50	0.062500
3	0.75	0.218750
4	1.00	0.515625

Problem: Find an approximate value of $\int_5^8 6x^3 dx$ using Euler's method of solving an ordinary differential equation. Use a step size of $h = 1.5$.

Solution: Given $\int_5^8 6x^3 dx$, we can rewrite the integral as the solution of an ordinary differential equation

$$\frac{dy}{dx} = 6x^3,\ y(5) = 0$$

where $y(8)$ will give the value of the integral $\int_5^8 6x^3 dx$.

$$\frac{dy}{dx} = 6x^3 = f(x, y),\ y(5) = 0$$

The Euler's method equation is $y_{i+1} = y_i + f(x_i, y_i)h$

Step 1: $i = 0,\ x_0 = 5,\ y_0 = 0\ \ h = 1.5 \quad x_1 = x_0 + h = 5 + 1.5 = 6.5$

$$\begin{aligned} y_1 &= y_0 + f(x_0, y_0)h \\ &= 0 + f(5,0) \times 1.5 \\ &= 0 + (6 \times 5^3) \times 1.5 \\ &= 1125 \\ &\approx y(6.5) \end{aligned}$$

Step 2: $i = 1,\ x_1 = 6.5,\ y_1 = 1125 \quad x_2 = x_1 + h = 6.5 + 1.5 = 8$

$$\begin{aligned} y_2 &= y_1 + f(x_1, y_1)h \\ &= 1125 + f(6.5, 1125) \times 1.5 \\ &= 1125 + (6 \times 6.5^3) \times 1.5 \\ &= 3596.625 \\ &\approx y(8) \end{aligned}$$

Hence

$$\int_5^8 6x^3 dx = y(8) - y(5) \approx 3596.625 - 0 = 3596.625$$

Problem: Using Euler's Method solve the following differential equation in four steps $\frac{dy}{dx} = x + y, \ \ y(0) = 0$ choosing h=0.2

Solution: Here $\frac{dy}{dx} = x + y \Rightarrow f(x, y) = x + y$

$$As \ \ y(0) = 0 \ \text{ so } x_0 = 0, \ y_0 = 0 \text{ and h=0.2}$$

By Euler's method -

$$y_{n+1} = y_n + hf(x_n, y_n)$$
$$y_1 = y_0 + hf(x_0, y_0) \Rightarrow y_1 = y_0 + h(x_0, y_0)$$
$$y_1 = 0 + (0.2)(0 + 0) = 0 \Rightarrow y_1 = 0$$
$$(x_1 = x_0 + h = 0 + 0.2 = 0.2)$$

$$y_2 = y_1 + h(x_1 + y_1) = 0 + (0.2)(0.2 + 0) = 0.04$$
$$y_3 = y_2 + h(x_2 + y_2) = 0.04 + (0.2)(0.4 + 0.04) = 0.04 + 0.088 = 0.128$$
$$(x_2 = x_1 + h = 0.2 + 0.2 = 0.4)$$
$$y_4 = y_3 + h(x_3 + y_3) = 0.128 + (0.2)(0.6 + 0.128) = 0.128 + 0.1456 = 0.2736$$

Improved Euler's method; In order to minimize the error between the solution and its approximate solution, the improved Euler method was developed.

Improved Euler's method: The approximate solution $Y_n = (y_1, y_2, y_3, ...,y_n)$ is defined by

$$y_{n+1} = y_n + h \, \frac{f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})}{2}$$

where $y^*_{n+1} = y_n + h \, f(x_n, y_n)$

Problem: Find solution of the initial value problem y' = 2x + y, y(0) = 1, on the interval $0 \leq x \leq 0.4$ by using four equal subintervals.

Solution: Dividing the interval [0, 0.4] into four equal parts, we get $h = \frac{0.4 - 0}{4} = 0.1$. Using f(x, y) = 2x + y and $x_0 = 0$, $y_0 = 1$, the required computation is conveniently arranged as follows:

Euler's Method for y' = 2x+y, y (0) =1

x_n	y_n	$y_n+0.1(2x_n+y_n) = y_{n+1}$
0	1.0	1.0+0.1[2(0)+1.0]=1.1
0.1	1.1	1.1+0.1[2(0.1)+1.1]=1.23
0.2	1.23	1.23+0.1[2(0.2)+1.23]=1.39
0.3	1.39	1.39+0.1[2(0.3)+1.39]=1.59
0.4	1.59	

Problem: Use the improved Euler method with h = 0.1 to estimate y (0.4), if y' = 2x+ y, y (0) =1. Compare the result with y (0.4) =1.6755.

Solution: The computations are as follows:

The Improved Euler Method

x_n	y_n	$y_t = y_n + 0.1(2x_n + y_n)$	$M = \frac{1}{2}[(2x_n + y_n) + (2x_{n+1} + y_t)]$	$y_{n+1} = y_n + 0.1M$
0	1	1.1	1.15	1.115
0.1	1.115	1.247	1.481	1.263
0.2	1.263	1.429	1.846	1.448
0.3	1.448	1.653	2.250	1.673
0.4	1.673			

Compared to the exact value of 1.6755, the percentage error is about 0.1% that the percentage error using the Euler method with h=0.1 is 5.4%.

Runge's Method (Second Order)

Euler's modified formula is

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1})\right]$$

$$\Rightarrow y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+h}, y_{n+h})\right]\text{.............}(1)$$

as $y_{n+1} = y_n + hfn$

Let $k_1 = hf(x_n, y_n)$

$\&$ $k_2 = hf(x_n + \text{h}, y_n + hf(x_n, y_n)$

$\Rightarrow k_2 = hf(x_n + \text{h}, y_n + k_1)$..............(1)

Putting the value of k_1 & k_2 we get

$$y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2)$$

Runge's formula of order 2.

Problem: Apply Runge's formula of 2nd order to find approximate value of y when x =1.1, given $\frac{dy}{dx} = 3x + y^2$ and $y=1.2$ when $x=1$.

Solution: Here, we have $x_0=1, y_0=1.2, h=0.1$

$$f(x,y) = 3x + y^2, f(x_0, y_0) = 3\times 1 + (1.2)^2 = 4.44$$

$$k_1 = hf(x_0, y_0) = 0.1\times 4.44 = 0.444$$

$$k_2 = hf(x_0 + \text{h}, y_0 + k_1) = 0.1(1.1, 1.2 + 0.444)$$

$$= 0.1f(1.1, 1.644)$$

$$= 0.1\left[3\times 1.1 + (1.644)^2\right]$$

$$= 0.600$$

$$y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2) = 1.2 + \frac{1}{2}(0.444 + 0.600) = 1.722$$

Runge's formula (Trird Order)

$$y_1 = y_0 + \frac{1}{6}(k_1 + 4k_2 + k_3)$$

where, $k_1 = hf(x_0, y_0), k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$$k_3 = hf(x_0 + h, y_0 + 2k_2 - k_1)$$

$$y_3 = y_0 + \frac{1}{6}(k_1 + 4k_2 + k_3)$$

Problem: Using Runge's formula (Third order) Solve the differential equation $\frac{dy}{dx} = x - y$ such that y = 1 when x = 1and find y (1.1).

Solution: $f(x, y) = x - y$, here $h = 0.1, x_0 = 1, y_0 = 1$

$$k_1 = hf(x_0, y_0) = 0.1(x - y) = 0.1(1-1) = 0$$

$$k_2 = hf\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right]$$

$$= 0.1f\left[1 + \frac{0.1}{2}, 1 + \frac{0}{2}\right]$$

$$= (0.1)f(1.05, 1)$$

$$= (0.1)(1.05 - 1) = 0.005$$

$$k_3 = hf(x_0 + \text{h}, y_0 + 2k_2 - k_1)$$

$$= 0.1f\left[1 + 0.1, 1 + 2(0.005) - 0\right]$$

$$= (0.1)f(1.1, 1.01)$$

$$= (0.1) \times (1.1 - 1.01) = 0.009$$

$$y_1 = y_0 + \frac{1}{6}(k_1 + 4k_2 + k_3)$$

$$y_{1.0} = 1 + \frac{1}{6}\left[0 + 4(0.005) + 0.009\right]$$

$$= 1 + \frac{1}{6}[0.02 + 0.009] = 1 + \frac{1}{6}[0.029] = 1.004833$$

So *y* at *x* = 1.1 is 1.004833.

Runge-Kutta Formula (Fourth Order)

Fourth order Runge-Kutta formula for solving the differential equation is

$$y = y_0 + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

where $k_1 = hf(x_0, y_0) \quad k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right), \quad k_4 = hf\left(x_0 + h, y_0 + k_3\right)$$

$$y = y_0 + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

Remark: Error in this formula is of order h^5 with greater accuracy.

Problem: Apply Runge Kutta method of fourth order to solve $5\frac{dy}{dx} = x^2 + y^2,\ y(0) = 1$ and find *y (0.2)* taking *h= 0.2*.

Solution: We have $5\frac{dy}{dx} = x^2 + y^2 \Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2}{5}$

$$\Rightarrow f(x_1 y_1) = \frac{x^2 + y^2}{5}$$

Let $h = 0.1,\ x_0 = 0,\ y_0 = 1$

$$k_1 = hf(x_0, y_0) = (0.1)f(0,1) = (0.1)\ \left(\frac{0+1}{5}\right) = 0.02$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = (0.1)f\left(0 + \frac{0.1}{2}, 1 + \frac{0.02}{2}\right)$$

$$= (0.1)f(0.05, 1.01) = (0.1)\left[\frac{(0.05)^2 + (1.01)^2}{5}\right] = 0.020452$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$=(0.1)f\left(0.05, \quad 1+\frac{0.020452}{2}\right)$$

$$=(0.1)f\left(0.05, \quad 1.01\right) = (0.1)\left[\frac{(0.05)^2 + (1.010226)^2}{5}\right] = 0.020461$$

$$k_4 = hf\left(x_0 + h, y_0 + k_3\right) = (0.1)f\left(0.1, \quad 1.020461\right)$$

$$= (0.1)\left[\frac{(0.1)^2 + (1.020461)^2}{5}\right] = 0.021027$$

$$y(0.1) = y_0 + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$=1 + \frac{1}{6}\left(0.02 + 2(0.020452) + 2(0.020461) + 0.021027\right)$$

$$=1+0.020476=1.020476$$

so $y(0.1) = 1.020476$ and $h = 0.1$

To calculate $y(0.2)$

$$k_1 = hf(x_1, y_1) = (0.1)f(0.1, 1.020476)$$

$$=(0.1)\left(\frac{(0.1)^2 + (1.020476)^2}{5}\right)=0.021027$$

$$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right) = (0.1)f\left(0.1 + \frac{0.1}{2}, 1.020476 + \frac{0.021027}{2}\right)$$

$$=(0.1)f(0.15, 1.030990) = (0.1)\left[\frac{(0.15)^2 + (1.030990)^2}{5}\right] = 0.021709$$

$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$=(0.1)\left(0.1, + \frac{0.1}{2}, 1.020476 + \frac{0.021709}{2}\right)$$

$$=(0.1)f\left(0.15, \quad 1.031331\right) = (0.1)\left[\frac{(0.15)^2 + (1.031331)^2}{5}\right] = 0.021723$$

$$k_4 = hf\left(x_1 + h, y_1 + k_3\right) = (0.1)f\left(0.1 + 0.1,\ \ 1.020476{+}0.021723\right)$$
$$= (0.1)\,f\,(0.2,1.042199)$$
$$= (0.1)\left[\frac{(0.2)^2 + (1.042199)^2}{5}\right] = 0.022524$$

$$so\ \ y(0.2) = y_1 + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$
$$=1.020476 + \frac{1}{6}\left(0.021027 + 2(0.021709) + 2(0.021723) + 0.022524\right)$$
$$=1.020476{+}0.021736{=}1.042212$$

Problem: Estimate y (1) if $2yy^1 = x^2$ and y (0) = 2 using Runge-Kutta method of fourth order by taking h = 0.5. Also compare the result with exact value.

Solution: Here h = 0.5, x_0=0, y_0=2, f(x, y) = $\dfrac{x^2}{2y}$.

$$k_1 = hf\,(x_0, y_0) = (0.5)\left(\frac{0}{4}\right) = 0$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = (0.5)\,f\left(0.25, 2\right) = \left(0.5\right)\times\frac{(0.25)^2}{4} = 0.0078$$
$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$
$$=(0.5)f\left(0.25,\ \ 2.0039\right) = \frac{(0.5)\times(0.25)^2}{2(2.0039)} = 0.0078$$
$$k_4 = hf\left(x_0 + h, y_0 + k_3\right) = (0.5)f\left(0 + 0.5,\ \ 2{+}0.0078\right)$$
$$= (0.5)\times\frac{(0.5)^2}{2(2.0078)} = 0.0311$$
$$so\ \ y(0.5) = y_0 + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$
$$=2 + \frac{1}{6}\left[0 + 2(0.0078) + 2(0.0078) + 0.03\right]{=}2.0104$$

For second step, x = 0.5, y (0.5) = 2.0104

$$k_1 = hf(x_{0.5}, y_{0.5})$$
$$= \frac{(0.5)\times(0.5)^2}{2(2.0104)} = 0.0311$$

$$k_2 = hf(0.75, 2.0156) = \frac{(0.5)\times(0.75)^2}{2(2.0156)} = 0.0698$$
$$k_3 = hf(0.75, 2.0349) = \frac{(0.5)\times(0.75)^2}{2(2.0349)} = 0.0691$$
$$k_4 = hf(1, 2.0691) = \frac{(0.5)\times(1)^2}{2(2.0691)} = 0.1208$$

According to Runge Kutta (Fourth order) formula

$$y(1) = y(0.5) + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$
$$=2.0104 + \frac{1}{6}\left[0.0311 + 0.1396 + 0.1382 + 0.1208\right]$$
$$=2.0104+0.0716=2.082$$

Exact value of y (1): Integrating $2yy^1 = x^2$ we get

$$y^2 = \frac{x^3}{3} + C$$

when x = 0 and y = 2, u = 0 + c

$$\Rightarrow c = 4$$

So putting *x=1*, we get

$$y^2(1) = \frac{1}{3} + 4 \Rightarrow y^2(1) = \frac{13}{4}$$

$$\Rightarrow y(1) = 2.08166 \text{ exact value}$$

Calculated value 2.082

Runge - Kutta Method for simultaneous Ist order Differential Equation.

Problem: Find y (0.1), z (0.1) from equation

$$\frac{dy}{dx} = x + z$$

$$\frac{dz}{dx} = x - y^2$$

y (0) = 2, z(0) = 1 using Runge - Kutta method of fourth order.

Solution: We have, $\frac{dy}{dx} = x + z,\ \frac{dz}{dx} = x - y^2$

So, $f_1(x, y, z) = x + z,\ f_2(x, y, z) = x - y^2$

$$x_0 = 0, y_0 = 2, z_0 = 1, h = 0.1$$

We use,

$$k_1 = hf_1(x_0, y_0, z_0) \qquad l_1 = hf_2(x_0, y_0, z_0)$$

$$k_2 = hf_1(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2})$$

$$l_2 = hf_2(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2})$$

$$k_3 = hf_1(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2})$$

$$l_3 = hf_3(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2})$$

$$k_4 = hf_1(x_0 + h, y_0 + k_3, z_0 + l_3) \qquad l_4 = hf_2(x_0 + h, y_0 + k_3, z_0 + l_3)$$

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \qquad \Delta z = \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$$

$$k_1 = (0.1) f_1(0, 2, 1) \qquad l_1 = (0.1) f_2(0, 2, 1)$$

$$=(0.1)(0+1)=0.1 \qquad =(0.1)(0-2^2)=-0.4$$

$k_2 = (0.1) f_1(0.05, 2.05, 0.81)$
=(0.1)(0.05+0.81)=0.085

$l_2 = (0.1) f_2(0.05, 2.05, 0.8)$
=(0.1)(0.05-(2.05)^2=-0.41525

$k_3 = (0.1) f_1(0.05, 2.0425, 0.79238)$
=(0.1)(0.05+0.79238)=0.084238

$l_3 = (0.1) f(0.05, 2.0425, 0.79238)$
=(0.1)[(0.05-(2.0425)^2]=-0.4122

$k_4 = (0.1) f_1(0.1, 2.084238, 0.5878)$
=(0.1)(0.1+0.5878)=0.06878

$l_4 = (0.1)[0.1 - (2.084238)^2]$
=-0.42214

$$so\ y_1 = 2 + \frac{1}{6}[0.1 + 2(0.085 + 0.084238) + 0.06878] = 2.0845$$

$$z_1 = 1 + \frac{1}{6}[-0.4 - (0.41525 + 0.4122) \times 2 - 0.4244]$$
$$= 0.5868$$

$so\ y(0.1) = 2.0845$ and $z(0.1) = 0.5868.$

Exercise

1. Using Picard's approximation, obtain a solution upto fifth approximation of the equation y'= y + x, y (0) =1. Compare your answer by finding exact solution.

2. Solve y'=y, y(0)=1 by Picard's method & compare the solution with exact solution.

3. Use Picard's method to obtain a solution upto 3rd order approximation of the equation y'=1+y^2. y(0)=0.

Exercise

4. Solve $y' = y - \frac{2x}{y}$, y (0) =1, h =.1 for $0 \le x \le .2$

Using (i) Euler's method (ii) Improved Euler's method

Apply the Euler method to approximate the indicated value of the solution function.

5. $y' = x+y$, y(0) =1, Find y(1) ,using h=.1

6. $y' = 1-y$, y(0) =0, Find y(.3), using h=.1

7. $y' = x^3+y$, y(0) =1. Find y(0.02) , using h=.01

8. $y' = x^2+y$, y(0) = 1, Find y (0.02),using h = .01

Apply the improved Euler method to approximate the indicated value of the solution function in following problems.

9. $y' = x^2+y$, y(0) =1, Find y(0.02), using h = .01

10. $y' = x+y$, y(0)=1, Find y(0.3), using h=.1

11. $y' = x+y^2$, y(0) =1, Find y(0.5), using h=.1

Given the initial-value problems, use the Runge Kutta method with h = 0.1 to obtain four decimal-place approximation to the indicated value.

12. $y' = x^2-y$, y(0) =1; y(0.1), y(0.2)

13. $y' = x^2+y^2$, y(1) = 1.5 ; y(1.2)

14. $y' = x+y^2$, y(0) =1; y(0.2)