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sign of faculty.

Subject: DSD UNIT-I Lecture 5 Mr. P. C. Yadav

Topic 1 → Gray code

Problem → convert $(10111011)_2$ in binary into Gray code.

Solution: Binary code

MSB $1 - \oplus \rightarrow 0 - \oplus \rightarrow 1 - \oplus \rightarrow 1 - \oplus \rightarrow 1 - \oplus \rightarrow 0 - \oplus \rightarrow 1 - \oplus \rightarrow 1$ LSB

$(1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0)_g$

$\therefore (10111011)_2 = (11100110)_{\text{Gray code}}$

Problem 2: convert gray code (101011) into binary code

Solution →

MSB $1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1$ LSB g_0

$1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0$ (B_0)

$(101011)_g = (110010)_2$

Boolean Algebra ⇒ The symbol which represent an arbitrary of an Boolean algebra is known as variable. Any single variable or a function of several variables can have either a 1 or 0 value. For example, in expression $Y = A + BC$, variables A, B and C can have eighter a 1 or 0 value, and function Y also can have either a 1 or 0 value. however its value depends on the value of Boolean expression.

Let $\begin{bmatrix} A = 0, & \bar{A} = 1 \\ A = 1, & \bar{A} = 0 \end{bmatrix}$

Fundamental Postulates of Boolean Algebra ②

* when two binary elements are operated by operator + the result is unique binary element.
* when two binary elements are operated by binary operator ·(dot), the result is a unique binary element.

Sr.No	Postulate	Comment
1	Result of each operation is either 0 or 1	$1, 0 \in B$
2	a) $0+0=0$ $0+1=1$ $1\cdot 1=1$ $1+0=1$ $1\cdot 0=0\cdot 1=0$	Identity elements 0 for + and 1 for ·
3	a) $A+B=B+A$ b) $A\cdot B=B\cdot A$	Commutative Laws
4	a) $A\cdot(B+C)=(A\cdot B)+(A\cdot C)$ b) $A+(B\cdot C)=(A+B)\cdot(A+C)$	Distributive Laws
5	a) $A+\bar{A}=1$, $0+\bar{0}=0+1=1$ and $1+\bar{1}=1+0=1$ b) $A\cdot\bar{A}=0$ since $0\cdot\bar{0}=0\cdot 1=0$ and $1\cdot\bar{1}=1\cdot 0=0$	Complement

Basic Theorems and properties of Boolean Algebra ⇒

1> **Duality** ⇒ The principle of duality theorem says that, starting with a Boolean relation, you can derive another Boolean relation by

1- changing each OR sign to an AND sign
2- changing each AND sign to an OR sign
3- complementing any 0 or 1 appearing in the expression

For example ⇒ Dual of relation $A+\overline{A}=1$ is
$A\cdot\overline{A}=0$

Duality is a very important property of Boolean Algebra.

Basic Theorems ⇒ ① $A+A=A$

$1+1=\overset{S}{0}\ \overset{C}{1}$ (Binary addition)

$$\begin{array}{|c|}\hline 0\\ 1\\ \hline\end{array}+\begin{array}{|c|}\hline 0\\ 1\\ \hline\end{array}=\begin{array}{|c|}\hline 0\\ 1\\ \hline\end{array}\Rightarrow A+A=A$$

$1+1=1\rightarrow$ Logic operation

Proof → $A+A=(A+A)\cdot 1=(A+A)\cdot(A+\overline{A})$
$\Rightarrow A+A\overline{A}=A+0=A$

② $A\cdot A=A$

Proof → $AA+0\Rightarrow AA+A\cdot\overline{A}$
$A(A+\overline{A})$ $\{A+\overline{A}=1\}$
$A\cdot 1=A$

③ $A+1=A$

④ $A\cdot 0=0$

⑤ $\overline{\overline{A}}=A$

⑥ $A+AB=A$ Important

Proof → $A+AB=A\cdot 1+AB$
$A(1+B)$ $\left\{\begin{array}{l}1+B=1\\ 1+A=1\end{array}\right\}$
$A\cdot 1=A$

⑦ $A(A+B)=A$
$=A\cdot A+AB$ $\{A\cdot A=A\}$
$=A+AB$ $\{A+AB=A\}$
$=A$

⑧ $A+\bar{A}B = A+B$

Proof ⇒ $A+\bar{A}B = A+AB+\bar{A}B$

$= A+B\cdot(A+\bar{A})$

$= A+B\cdot 1 = (A+B)$

⑨ $A\cdot(\bar{A}+B) = AB$

$= (A+AB)\cdot(\bar{A}+B)$

$= A\bar{A}+AB+ABB$

$= AB+ABB$

$= AB+AB$

$= \underline{AB}$ Proof

Demorgan's Theorem ⇒ Demorgan suggested two theorems that form an important part of Boolean algebra. In the equation form, they are

1) $\overline{AB} = \bar{A}+\bar{B}$ [The complement of a product is equal to the sum of the complements]

2) $\overline{A+B} = \bar{A}\cdot\bar{B}$ [The complements of a sum is equal to the product of the complements]

A	B	$\overline{A+B}$	$\bar{A}\bar{B}$
0	0	$\overline{0+0}=1$	1
0	1	$\overline{0+1}=0$	0
1	0	$\overline{1+0}=0$	0
1	1	$\overline{1+1}=0$	0

Consensus Theorem ⇒

$AB+\bar{A}C+BC = AB+\bar{A}C$

Proof ⇒ $AB+\bar{A}C+(A+\bar{A})BC$

⇒ $AB+\bar{A}C+AB+\bar{A}C$

⇒ $AB+AB+\bar{A}C+\bar{A}C$

$= AB+\bar{A}C$ $\left\{\begin{array}{l} AB+AB=AB \\ \bar{A}C+\bar{A}C=\bar{A}C \end{array}\right\}$

⇒ $\underline{AB+\bar{A}C}$ Proof

Dual of Consensus Theorem

$(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$

Proof → multiply each element on both sides, we get

$(\bar{A}B+BC+AC) = AC+\bar{A}B+BC$

$(\bar{A}B+BC+AC) = AC+\bar{A}B+BC$

Boolean Expression → $f(A,B,C) = (A+\bar{B})C$ or $(A+\bar{B})C$

Product terms

$f(A,B,C,D) = \boxed{A} + \boxed{\bar{B}C} + \boxed{AC\bar{D}}$

Literals

Sum of Product (SOP) Form

② Product of sum (POS form) ⇒

Sum terms

$f(A,B,C,D) = (B+\bar{D}) \cdot (A+\bar{B}+C) \cdot (\bar{A}+C)$

Literals

Standard SOP Form or Minterm canonical form ⇒

If each term in SOP form contains all the literals then the SOP form is known as standard or canonical SOP form. Each individual term in the standard SOP form is called minterm. Therefore, canonical SOP form is also known as minterm canonical form.

$f(A,B,C) = A\bar{B}C + ABC + \bar{A}B\bar{C}$

(Each product term consists of all literals in either complemented form or uncomplemented form)

Standard POS form or Maxterm canonical form.

- If each term in POS form contains all the literals then POS form is known as standard POS form. ~~is called max~~ Each individual term in the standard POS form is called maxterm.

$$f(A,B,C) = (\underbrace{A+B+C}) \cdot (\underbrace{A+\overline{B}+C})$$

(Each sum term consists of all literals in either complemented form or uncomplemented form)

Problem 1 ⇒ convert the given expression in standard SOP form.

$$f(A,B,C) = AC + AB + BC$$

Solution ⇒ Step 1 → find the missing literals in each product term.

$f(A,B,C) = AC + AB + BC$

BC → Literal A is missing

AB → Literal C is missing

AC → Literal B is missing

$$f(A,B,C) = AC(B+\overline{B}) + AB\cdot(C+\overline{C}) + BC(A+\overline{A})$$

$$= ACB + AC\overline{B} + ABC + AB\overline{C} + BCA + BC\overline{A}$$

$$f(A,B,C) = ABC + A\overline{B}C + ABC + AB\overline{C} + ABC + \overline{A}BC$$ ~~Ans~~

$$f(A,B,C) = ABC + A\overline{B}C + AB\overline{C} + \overline{A}BC$$ Ans

Problem 2 ⇒ convert the given expression in standard POS form.

$f(A,B,C) = (A+B)(B+C)(A+C)$

Solution:

$f(A,B,C) = \underbrace{(A+B)}_{\text{literal C is missing}}\underbrace{(B+C)}_{\text{A missing}}\underbrace{(A+C)}_{\text{B is missing}}$

$= \cancel{(A+B+C\bar{C}) \cdot (A+B+}$

$= (A+B+C\bar{C})(B+C+A\bar{A})(A+C+B\bar{B})$

$= (A+B+C)(A+B+\bar{C})(B+C+A)(B+C+\bar{A})(A+C+B)(A+C+\bar{B})$

$\therefore f = (A+B+C)(A+B+\bar{C})(\bar{A}+B+C)(A+\bar{B}+C)$

Ans

M-Notations: Minterm and Maxterms

Variables			minterms m_i	Maxterms M_i
A	B	C	m_i	M_i
0	0	0	$\bar{A}\bar{B}\bar{C} = m_0$	$A+B+C = M_0$
0	0	1	$\bar{A}\bar{B}C = m_1$	$A+B+\bar{C} = M_1$
0	1	0	$\bar{A}B\bar{C} = m_2$	$A+\bar{B}+C = M_2$
0	1	1	$\bar{A}BC = m_3$	$A+\bar{B}+\bar{C} = M_3$
1	0	0	$A\bar{B}\bar{C} = m_4$	$\bar{A}+B+C = M_4$
1	0	1	$A\bar{B}C = m_5$	$\bar{A}+B+\bar{C} = M_5$
1	1	0	$AB\bar{C} = m_6$	$\bar{A}+\bar{B}+C = M_6$
1	1	1	$ABC = m_7$	$\bar{A}+\bar{B}+\bar{C} = M_7$

Minterms and maxterms for three variables

1. $f(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + AB\bar{C}$

$= m_0 + m_1 + m_3 + m_6$

$= \Sigma m(0,1,3,6) \longrightarrow$ SOP form

2. $f(A,B,C) = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)$

$= M_1 + M_3 + M_6$

$f(A,B,C) = \pi M(1,3,6) \longrightarrow$ POS form

Where Σ denotes sum of Product while π ~~depends~~ denotes product of sum.

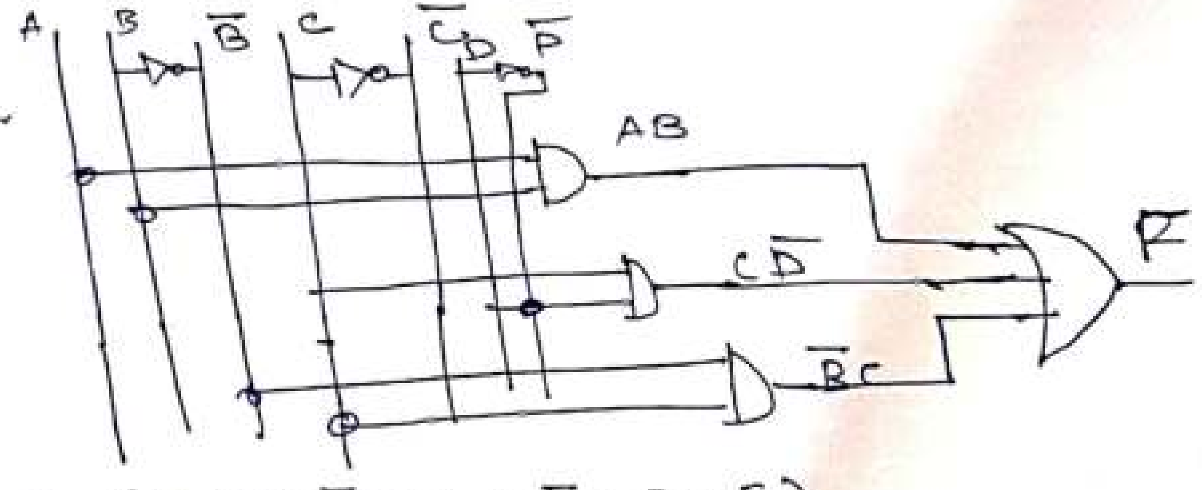
To solve Problems ⇒ (i) $ABC + A\bar{B}C + AB\bar{C}$

(ii) $AB + \overline{AC} + A\bar{B}C(AB+C)$ (iii) $AC + C(A + \bar{A}B)$

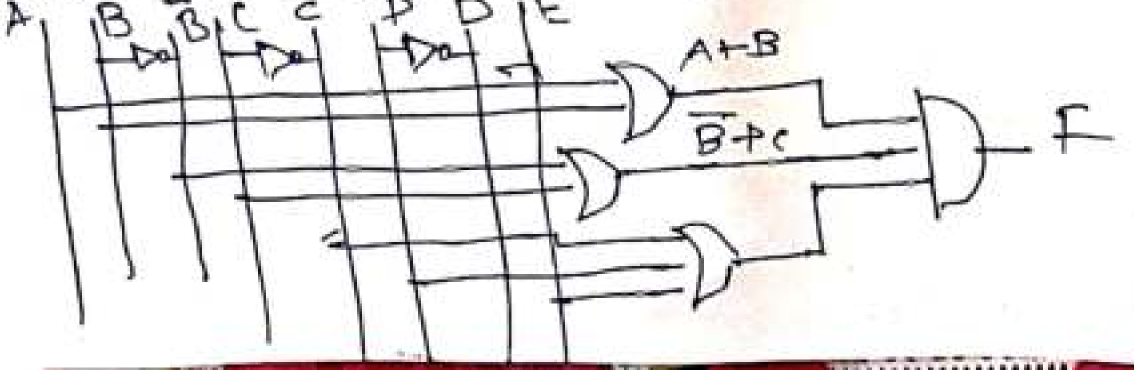
(iv) $\overline{\overline{AB} + \bar{A} + AB}$ (v) $\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC$

Implementation of SOP Boolean Expression ⇒

$F = AB + C\bar{D} + \bar{B}C$



POS → $F = (A+B)(\bar{B}+C)(\bar{C}+D+E)$



Number system UNIT - II

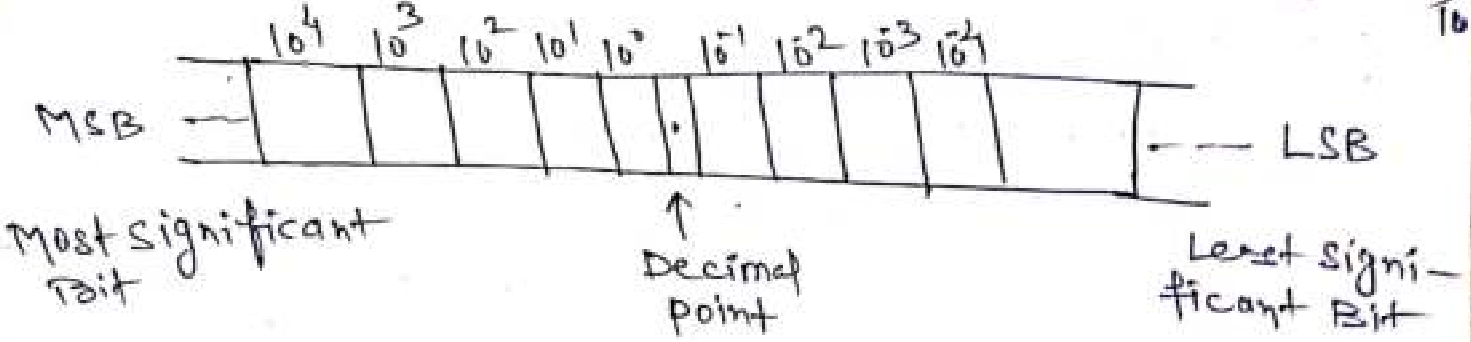
① Decimal Number system ⇒

$5000 + 600 + 70 + 8 + 0.9 = 5678.9$

Also written as $(5678.9)_{10}$

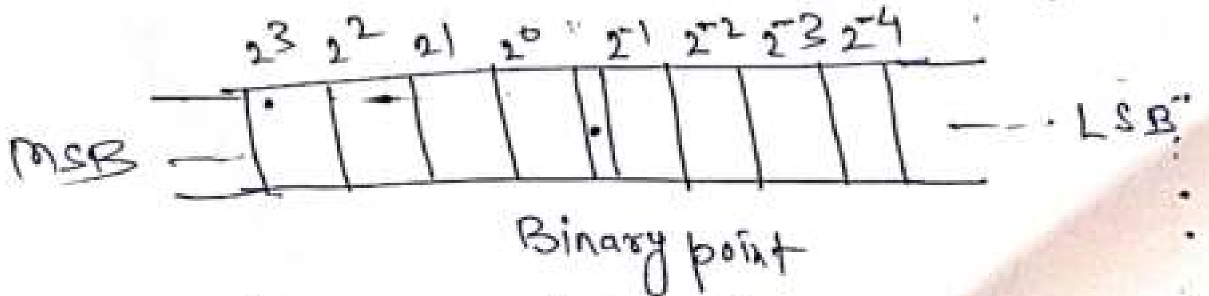
In power of 10, we can write as

$(5\times10^3 + 6\times10^2 + 7\times10^1 + 8\times10^0 + 9\times10^{-1}) = (5678.9)_{10}$



Decimal position values as power of 10

② Binary Number system ⇒ (0 or 1)



Binary position values as a power of 2

Example: Represent binary number 1101.101 in power of 2 and find its decimal.

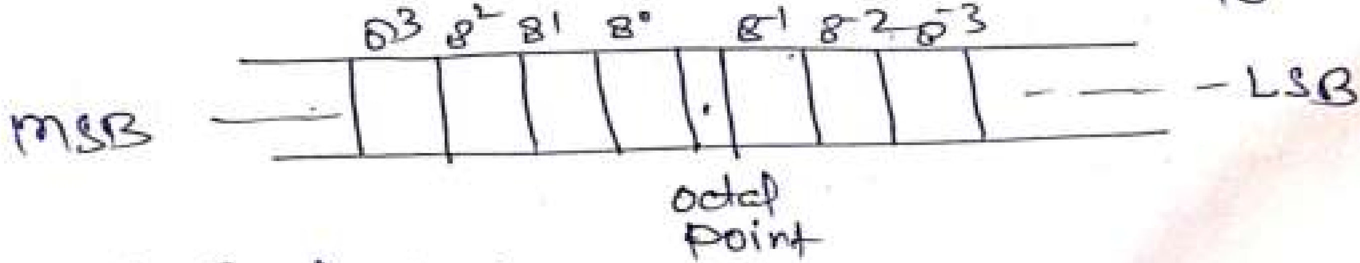
$N = 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0 + 1\times2^{-1} + 0\times2^{-2} + 1\times2^{-3}$

$= 8 + 4 + 0 + 1 + 0.5 + 0 + 0.125$

$N = (13.625)_{10}$

Octal Number system = $(\;)_8$

$$567 \Rightarrow (5\times8^2 + 6\times8^1 + 7\times8^0)$$
$$= 5\times64 + 6\times8 + 7\times1$$
$$= 320 + 48 + 7 = (375)_{10}$$



Hexadecimal Number system ⇒

Decimal	Binary	Hexadecimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Example: Represent hexadecimal number 3FD in power of 16 and find its decimal equivalent

$3\times16^2 + F\times16^1 + D\times16^0$ $\left\{ \begin{matrix} F = 15 \\ D = 13 \end{matrix} \right\}$

$= 3\times256 + 15\times16 + 13\times1$

$= 768 + 240 + 13 = (1021)_{10}$

Counting in Radix (Base) r ⇒

Radix (Base) r	characters in set
2 (Binary)	0,1
3	0,1,2
4	0,1,2,3
⋮	
7	0,1,2,3,4,5,6
8 (octal)	0,1,2,3,4,5,6,7
⋮	
10	0,1,2,3,4,5,6,7,8,9
⋮	
16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

Number system conversion

① Decimal Number into Binary number system ⇒

Ex → $(37.8125)_{10} = (\quad)_2$

Integer part = 37

Fractional part = 0.8125

2	37	
2	18	1
2	9	0
2	4	1
2	2	0
2	1	0
	0	1

$(100101)_2$

Fraction Radix = Result

MSD

0.8125 × 2 = 1.625 = 0.625 with a carry of 1

0.625 × 2 = 1.25 = 0.25 with a carry of 1

0.25 × 2 = 0.5 = 0.5 with carry of 0

0.50 × 2 = 1.0 = 0.0 with carry of 1

LSD

$(.8125)_{10} = (0.1101)_2$

II) Convert Decimal into octal Number

$(214)_{10} = (\ )_8 = (326)_8$

Quotient Reminder

8	214	
8	26	6
8	3	2
	0	3

↑ LSD

MSD

8	214	6
8	26	2
8	3	3
	0	

↑ LSD

MSD

III) Convert Decimal into Hexadecimal No

$(3509)_{10} = (\ )_{16} = (DB5)_{16}$

16	3509	5
16	219	11 → B
16	13	13 → D
	0	

↑ LSD

MSD

$= (DB5)_{16}$

Ex: $(0.640625)_{10} = (\ )_8 = (0.51)_8$

0.640625 × 8 = 5.125 = 0.125 with a carry of 5

0.125 × 8 = 1.00 = 0 with a carry of 1

- convert Binary into octal and Hexadecimal Number ⇒ ⑤

$$(1111000.001)_2 = (\ \ )_8 = (\ \ )_{16}$$

$$\overleftrightarrow{001}\,\overleftrightarrow{111}\,\overleftrightarrow{000}\,.\,\overleftrightarrow{001}$$

$$(1\ 7\ 0\ .\ 1) = (170.1)_8$$

$$(1111000.001)_2 = (78.2)_{16}$$

$$\overleftrightarrow{0111}\,\overleftrightarrow{1000}\,.\,\overleftrightarrow{0010}$$

$$(7\ 8\ .\ 2)_{16}$$

Problems: $(125)_\alpha = \cancel{(50)_{10}}\ (29)_{10}$

$$1\times\alpha^2 + 2\times\alpha^1 + 5\times\alpha^0 = \cancel{50}\ 29$$

$$\alpha^2 + 2\alpha + 5 = \cancel{50}\ 29$$

$$\cancel{\alpha^2 + 2\alpha - 45 = 0}$$

$$\alpha^2 + 2\alpha - 24 = 0$$

$$\alpha^2 + 6\alpha - 4\alpha - 24 = 0$$

$$\alpha(\alpha+6) - 4(\alpha+6) = 0$$

$$(\alpha-4)(\alpha+6) = 0$$

$$\alpha = 4, -6 \quad \underline{\text{Ans}}$$

Binary addition

	S	C
0+0 =	0	0
0+1	1	0
1+0	1	0
1+1	0	1

Binary subtraction

	D	B
0−0	0	0
0−1	1	1
1−0	1	0
1−1	0	0

Binary multiplication

$0 \times 0 = 0$
$0 \times 1 = 0$
$1 \times 0 = 0$
$1 \times 1 = 1$

Binary Division

$0 \div 0 = a.$
$0 \div 1 = 0$

$\frac{1}{1} = 1$

Octal Addition

i) $(4+2)_8 = (6)_8$

ii) $(6+7)_8 = (13-8)_8 = 5$ carry 1

iii) $(1+7)_8 = (8-8)_8 = 0$ carry 1

Example ⇒

$$\begin{array}{ccc} 1 & 1 & \\ 1 & 6 & 7 \\ 3 & 2 & 5 \\ \hline (5 & 1 & 4)_8 \end{array}$$

No carry simply added

$12-8=4$ carry 1

$9-8=1$ carry 1

ii) Hexadecimal Addition ⇒

Add. $(3F8)_{16}$ and $(5B3)_{16}$

$$\begin{array}{ccc} 1 & & \\ 3 & F & 8 \\ 5 & B & 3 \\ \hline 9 & (26-16) & B \\ & \Downarrow & \\ & A & \end{array} \Rightarrow (9AB)_{16}$$

Ans

Binary codes

when numbers, alphabets or words are represented by a specific group of symbols, we can say that they are encoded. The group of symbols used to encode them are called codes. The digital data is represented, stored and transmitted as groups of binary digits (bits). The group of bits also known as binary code. represent both numbers and letters of the alphabets as well as many special characters and control functions.

BCD (Binary coded Decimal) codes ⇒

Decimal Digit	BCD code 8	4	2	1
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1

Ex

$$\begin{array}{cc} 5 & 8 \\ \downarrow & \downarrow \\ 0101 & 1000 \end{array}$$

8421

BCD code