Control System

Mathematical Modeling
↓
i) Translational Motion
ii) Rotational Motion

For analyzing any physical system & after drawing free body diagram calculation of transfer function is known as Mathematical Modelling.

Q.

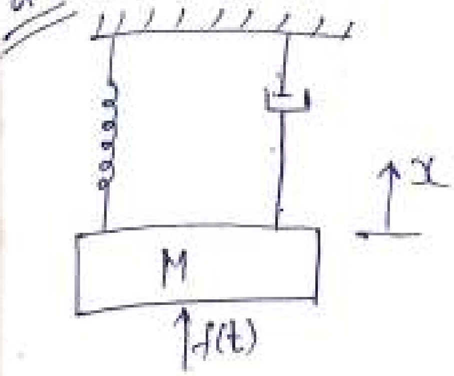
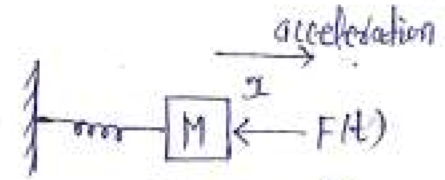
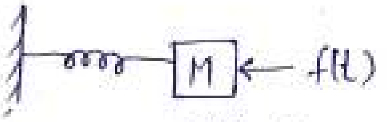


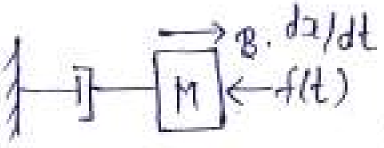
fig. 1



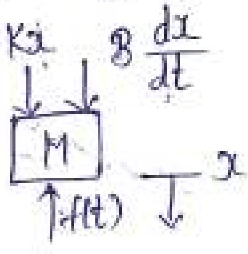
$f(t) = m\frac{d^2x}{dt^2}$



$f(t) = Kx$



free Body diagram



$$f(t) = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx \quad \text{--- (1)}$$

Taking the Laplace transform

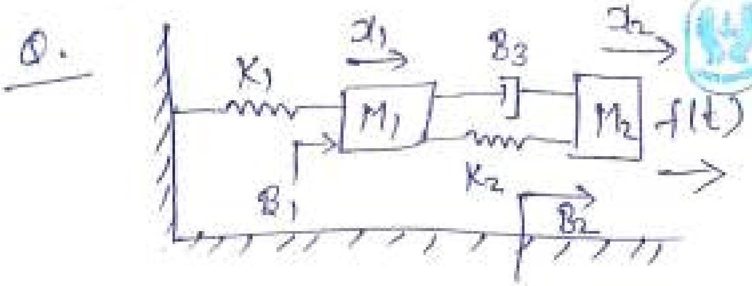
$$F(s) = Ms^2X(s) + KX(s) + BsX(s)$$

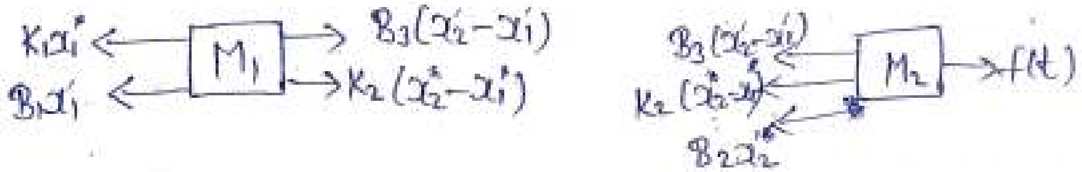
(Assuming initial condition are zero)

$$F(s) = (Ms^2 + K + Bs)X(s)$$

$$\frac{X(s)}{F(s)} = \left(\frac{1}{Ms^2 + K + Bs}\right)$$

Q.

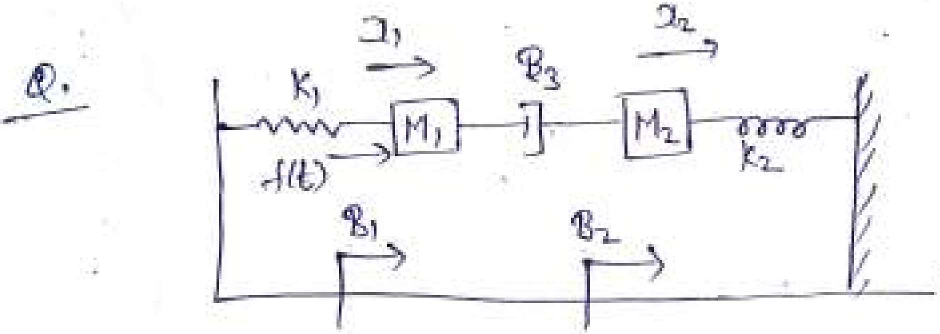




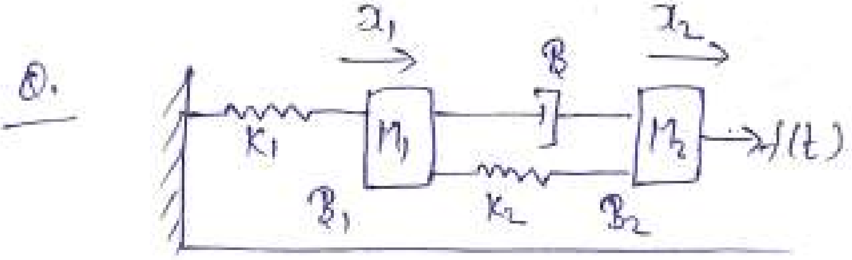
$$f(t) = M_2 x_2'' + B_2 x_2' + K_2(x_2 - x_1) + B_3(x_2' - x_1') \quad ①$$

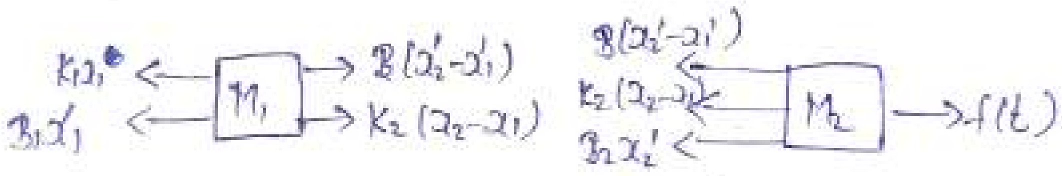
$$B_3(x_2' - x_1') + K_2(x_2 - x_1) = B_1 x_1' + K_1 x_1 + M_1 x_1''$$

Q.



Q.

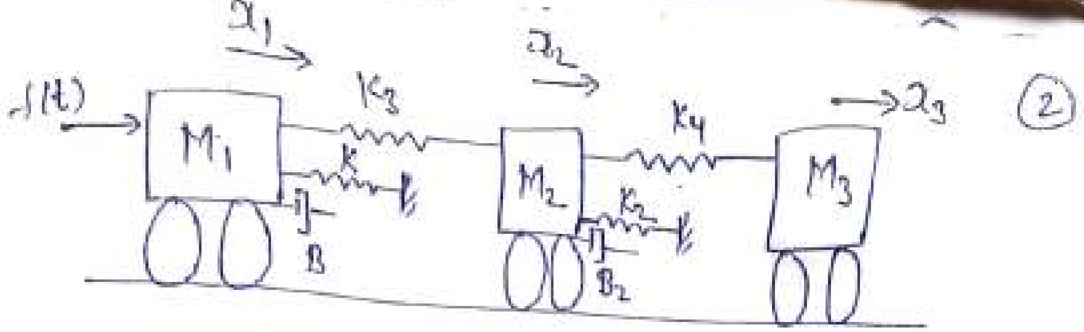


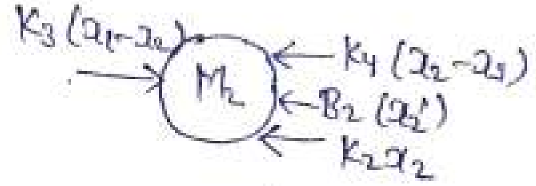


$$f(t) = M_2 x_2'' + B(x_2' - x_1') + K_2(x_2 - x_1) + B_2 x_2'$$

$$B_2(x_2' - x_1') + K_2(x_2 - x_1) = K_1 x_1 + B_1 x_1' + M_1 x_1''$$

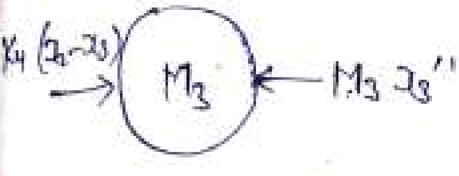
Q.





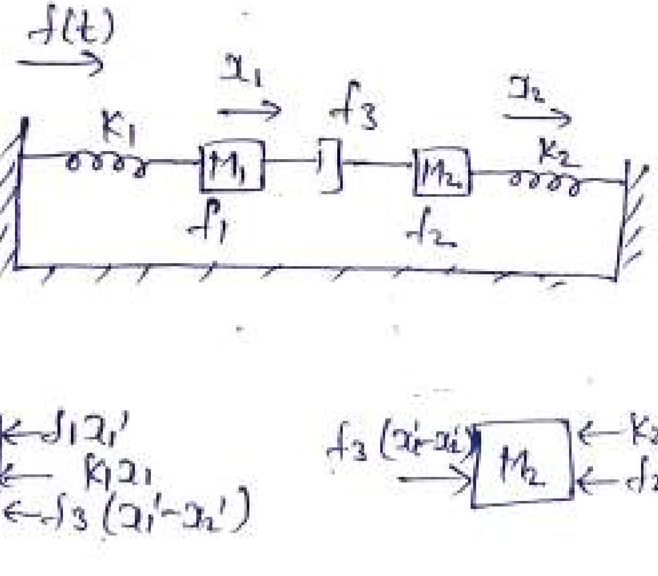
$f(t) = K_3(x_1-x_2) + K_1x_1 + B_1x_1' + M_1x_1''$

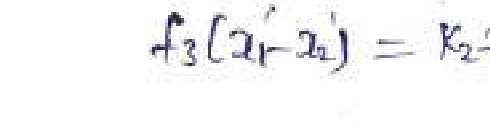
$K_3(x_1-x_2) = K_4(x_2-x_3) + B_2x_2' + K_2x_2 + M_2x_2''$ — (ii)

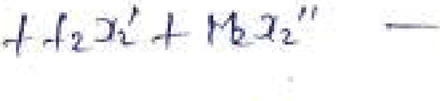


$M_3x_3'' = K_4(x_2-x_3)$ — (iii)

Q.
IES
05







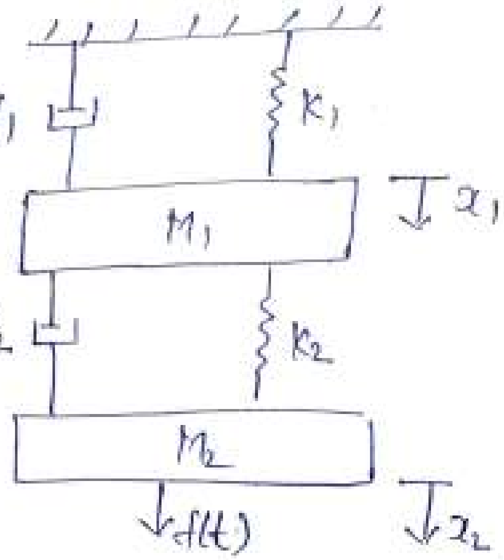
$f(t) = M_1x_1'' + K_1x_1 + f_1x_1' + f_3(x_1'-x_2')$ — (i)

$f_3(x_1'-x_2') = K_2x_2 + f_2x_2' + M_2x_2''$ — (ii)

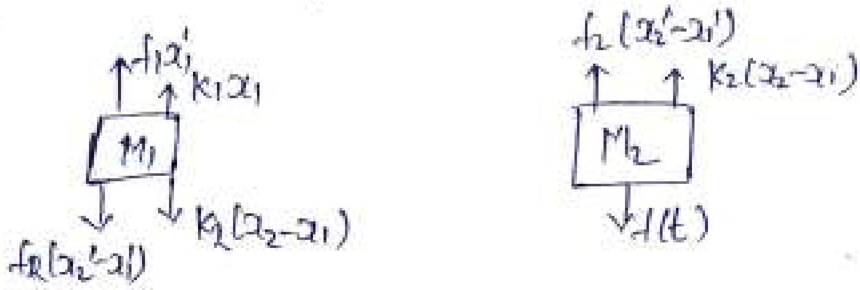
Note:- (i) Always start from External force

(ii) All forces are applied opposite to External force.

Q.



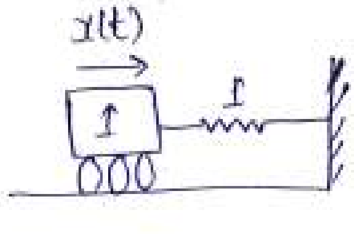
Sol.ⁿ



$$f(t) = M_2 x_2'' + f_2(x_2' - x_1') + K_2(x_2 - x_1)$$

$$f_2(x_2 - x_1) + K_2(x_2 - x_1) = M_1 x_1'' + K_1 x_1 + f_1 x_1'$$

Q. GATE



a) $x(t) = \sin t$

b) $x(t) = \sqrt{2} \sin t$

c) $x(t) = \sin \sqrt{2} t$

d) $x(t) = \frac{1}{2} \sin 2t$

Consider a Mechanical System shown in fig. If system is set into motion by unit impulse force. What is Equation of result oscillation.

Sol.ⁿ

$$f(t) \longrightarrow \boxed{M_1} \leftarrow Kx.$$

[Note:- As displacement in Right direction therefore force is also in right direction]

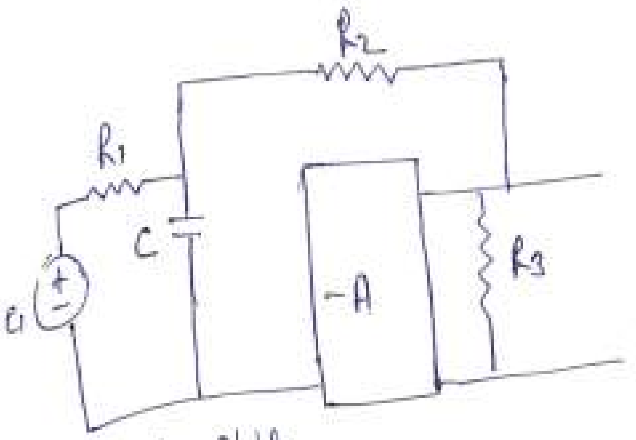
$f(t) = Mx'' + Kx$

taking laplace transform

$f(s) = (Ms^2 + K) X(s)$

$1 = (s^2 + 1) X(s)$

$X(s) = 1/(s^2+1) = \sin t$. Hence (a) is correct.



upper n/w is stable

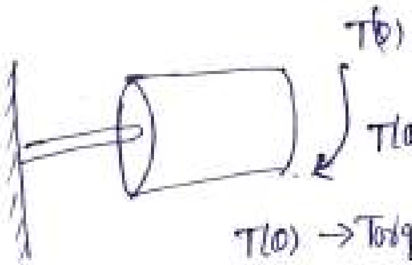
a) for all value of R & C (✓)

b) Is stable if $R_1R_2 = C$

c) Is stable if $R_1C = R_2R_3$

d) Is stable for $\frac{R_1}{R_2} = CR_3$

Rotational - Mechanical System



$$T(\theta) = J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} + K\theta$$

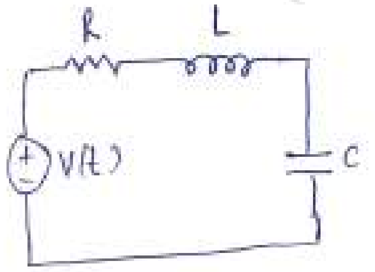
$T(\theta) \rightarrow$ Torque $\rightarrow$ force

$J \rightarrow$ M.I $\rightarrow$ Mass

f

$\theta \rightarrow x$

force-voltage Analogy →



$$v(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i\,dt \quad -①$$

$$f(t) = m\frac{d^2x}{dt^2} + Kx + B\frac{dx}{dt} \quad -②$$

$$i(t) = \frac{dq}{dt}$$

$V = L\frac{di}{dt}$

$i = \frac{1}{L}\int v\,dt$

$V = \frac{1}{C}\int_{-\infty}^{t} i\,dt$

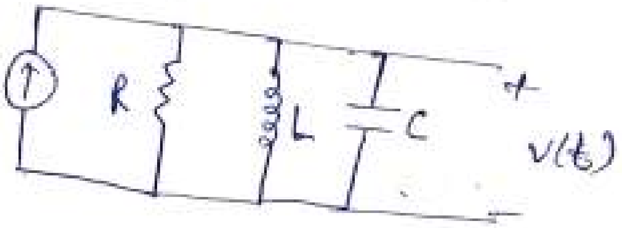
$i = C\frac{dv}{dt}$

Q.

$$v(t) = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q \quad \text{— (2)}$$

compare (2) & (3)

1) $f(t) \to v$
2) $x \to q$
3) $m \to L$
4) $B \to R$
5) $k \to 1/C$



$$I(t) = \frac{v}{R} + \frac{1}{L}\int_0^T v\,dt + C\frac{dv}{dt} \quad \text{— (1)}$$

$$f(t) = m\frac{d^2x}{dt^2} + Kx + f\frac{dx}{dt} \quad \text{— (3)}$$

Put $v = \frac{d\phi(t)}{dt}$ — in (1), we get

$$I(t) = \frac{1}{R}\frac{d\phi(t)}{dt} + \frac{1}{L}\phi(t) + C\frac{d^2\phi(t)}{dt^2} \quad \text{— (2)}$$

Compare (2) & (3), we get

$f(t) = I$

force $\to$ current

$x \to$ flux

$m \to C$

$K \to 1/L$

$f \to 1/R$

$X(s) = \ldots$

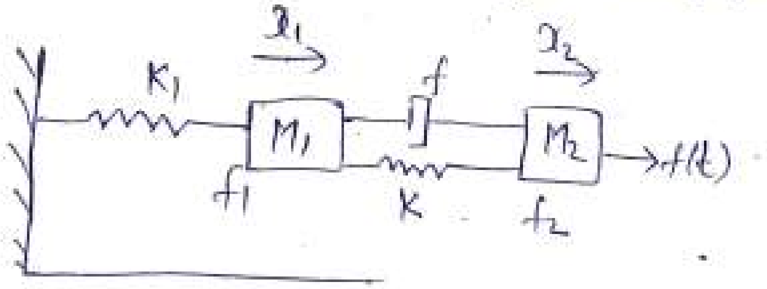
Comparison of

	F-V	F-I
1) force	V	I
2) x	q	ψ
3) Mass	L	C
4) K	1/C	1/L
5) f	R	1/R

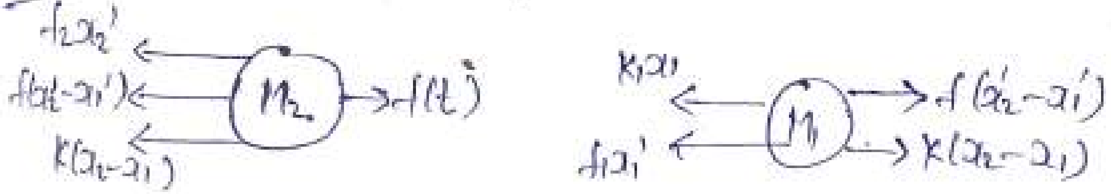
from these result it is clear that force-voltage & force-current are dual to each other i) Dual of inductance is capacitor & vice-versa.

ii) Dual of Resistance is inverse of resistance.

Q. Design electrical n/w for given mathematical model

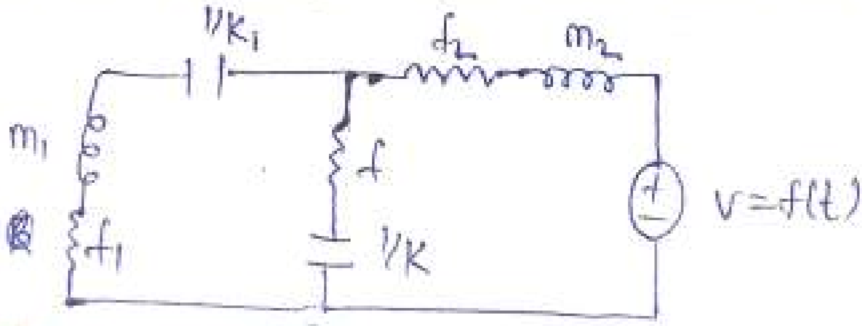


Sol^n

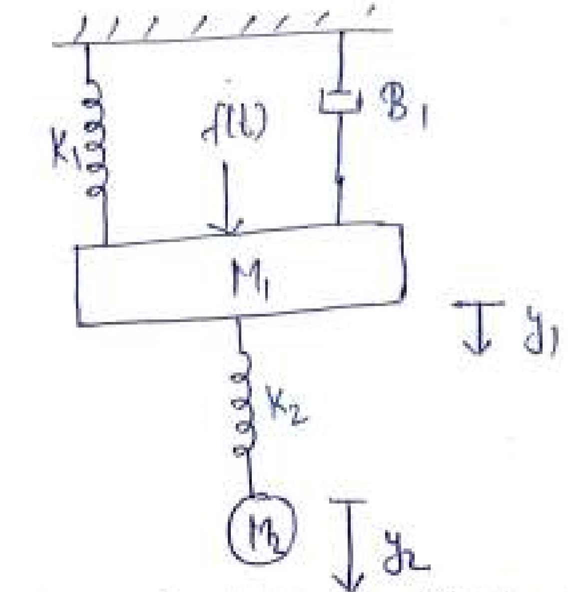


$$f(t) = m_2x_2'' + f_2x_2' + f(x_2'-x_1') + K(x_2-x_1) \quad \text{--- (1)}$$

$$f(x_2'-x_1') + K(x_2-x_1) = K_1x_1 + f_1x_1' + m_1x_1'' \quad \text{--- (2)}$$



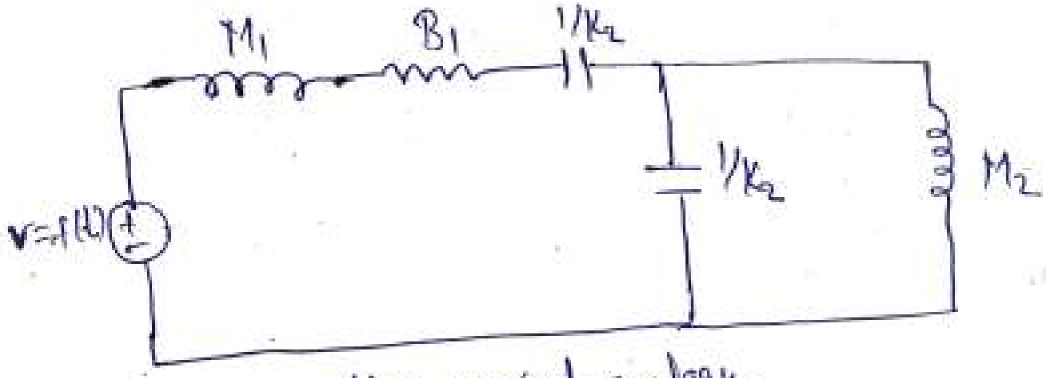
Q.



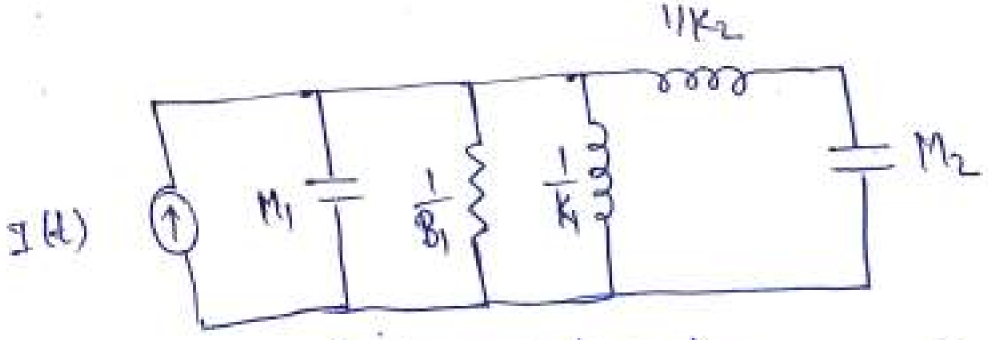
$I(t) = C\frac{d^2 j}{dt^2} + \frac{1}{R}\frac{dj}{dt} + \frac{1}{L}j$

$f(t) = M_1 y_1'' + K_1 y_1 + B_1 y_1' + K_2 (y_1 - y_2)$

$K_2 (y_1 - y_2) = M_2 y_2''$



voltage current analogy.



force-current analogy