$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2}$$

$$|e^{-j\theta}| = 1$$

Different type of functions:-

1. Step f'n
2. Ramp
3. Impulse
4. Triangular

Q. which is unit step f'n

1) $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

3. $u(t) = \begin{cases} 1, & t > 0 \\ 0, & t \leq 0 \end{cases}$

2. $u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$

4. $u(t) = \begin{cases} 1, & t > 0 \\ 1/2, & t = 0 \\ 0, & t < 0 \end{cases}$

A) 1, 3, 4

B) 1, 4

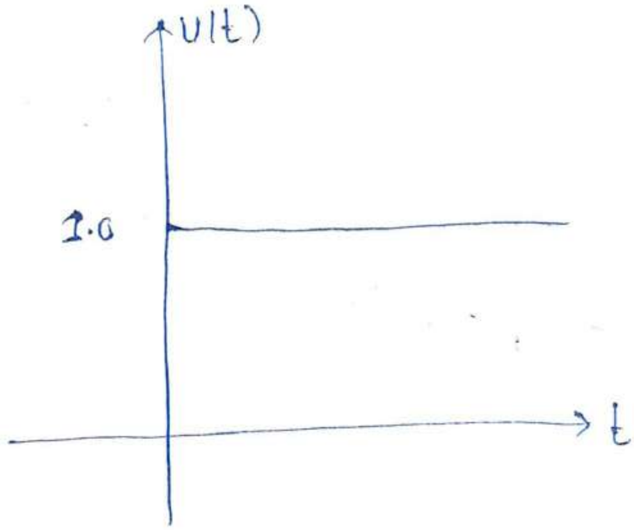
c) 1, 3

d) 1, 2, 3, 4 (✓)

this concept is valid for continuous time system & not valid for discrete time.

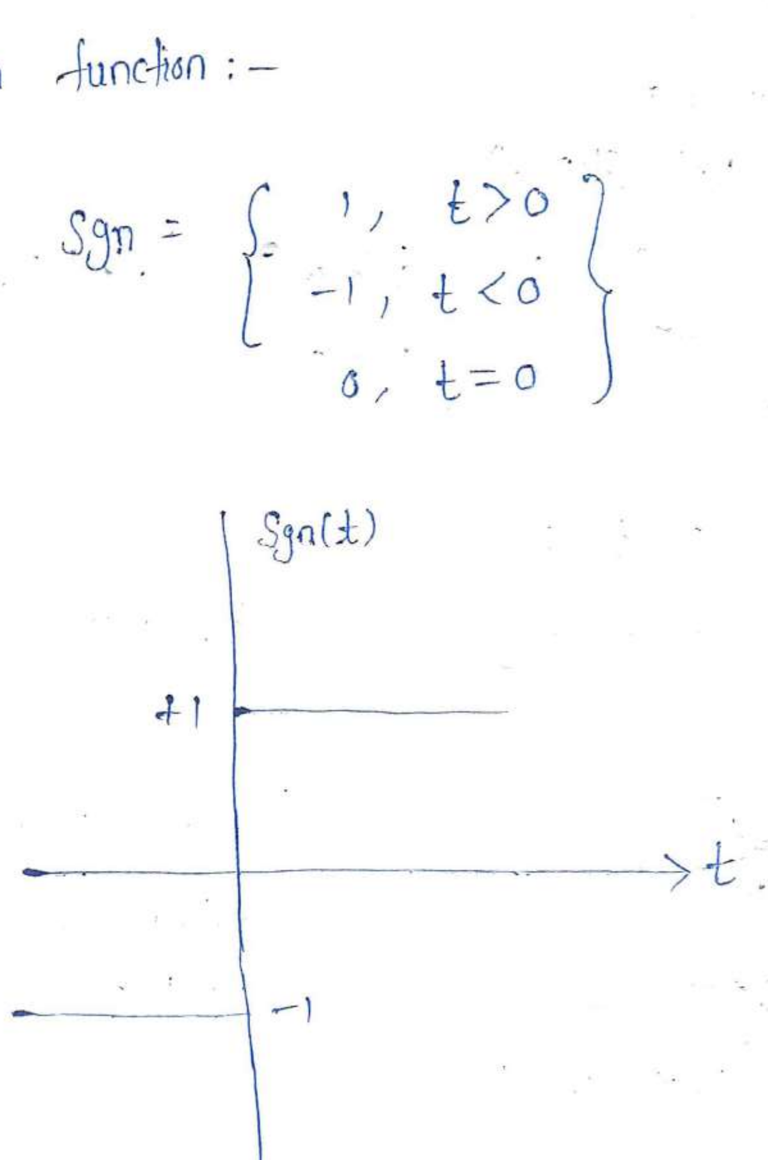
Ans. In this case all options are correct because a finite difference b/w two signal at one pt. has no real consequence on response of system for these two signal.

2)



2) Signum function :-

$$Sgn = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \\ 0, & t = 0 \end{cases}$$



Q. Sgn(t) is Equal to

a) $2u(t)$ b) $u(t)-1$

c) $2\{u(t)-1$ (✓) d) $2u(t)+1$

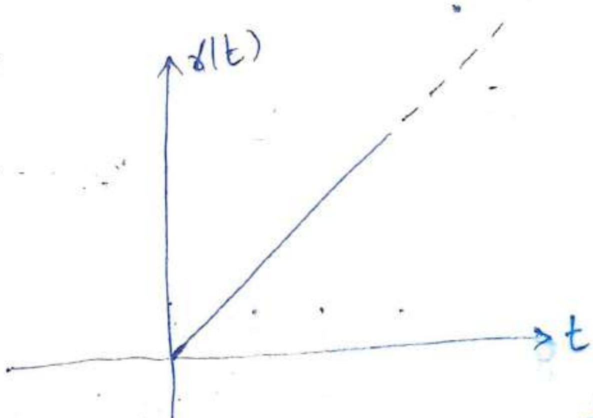
$$Sgn(t) = 2u(t) - 1$$

$$\boxed{u(t) = \frac{1 + Sgn(t)}{2}}$$

$$\begin{tikzcd} \delta(t) \arrow[d, "\int"] \\ u(t) \arrow[u, bend left, "d/dt"] \arrow[d, "\int"] \\ \text{Ramp} \arrow[u, bend left, "d/dt"] \arrow[r, "\int"] & \text{parabolic} \arrow[l, bend left, "d/dt"] \end{tikzcd}$$

Ramp function :-

$$r(t) = \begin{cases} t, & \text{for } t > 0 \\ 0, & \text{for } t < 0 \end{cases}$$



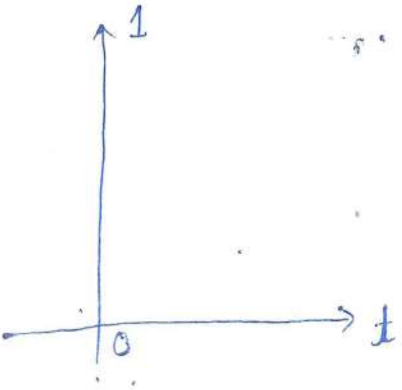
$$r(t) = \int_{-\infty}^{t} u(t)\,dt = \int_{-\infty}^{0} u(\tau)\,d\tau + \int_{0}^{t} u(t)\,dt$$

$$= 0 + \int_{0}^{t} u(\tau)\,d\tau$$

$$= t\,u(t)$$

Impulse :-

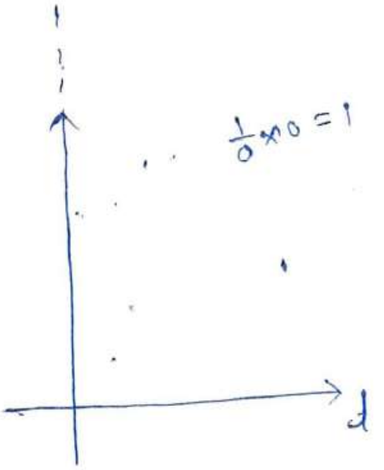
1)



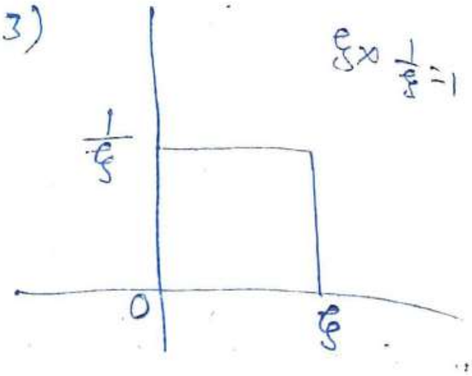
$= 1 \times 0$

$= 0$

2)



3)



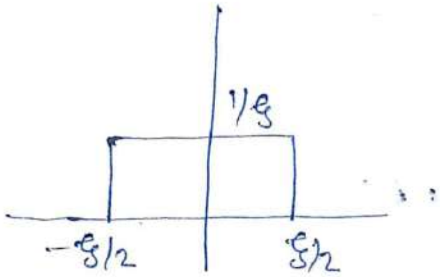
A) 2, 3, 4 (✓)

B) 1, 2, 3, 4

C) 1, 2

d) 1, 3

4)

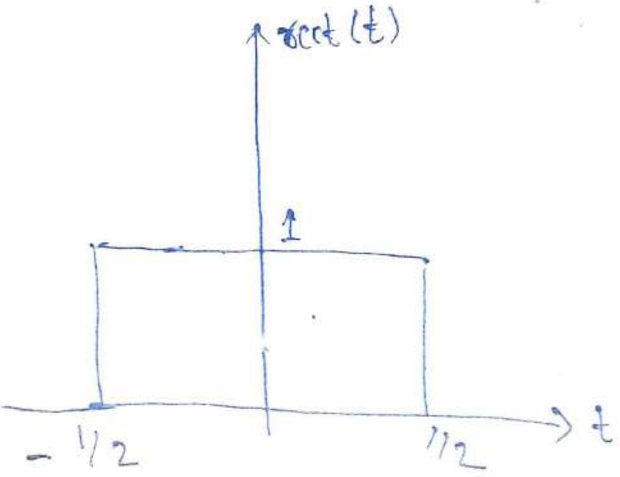


$1/\xi \times \xi = 1$

$$\xi = \begin{cases} 1, & t = 0 \\ 0 & \text{elsewhere} \end{cases}$$

~~A)~~ unit Impulse means area of ~~that~~ impulse f'n must be unity it does not mean ht. of ~~..~~ f'n is unity.

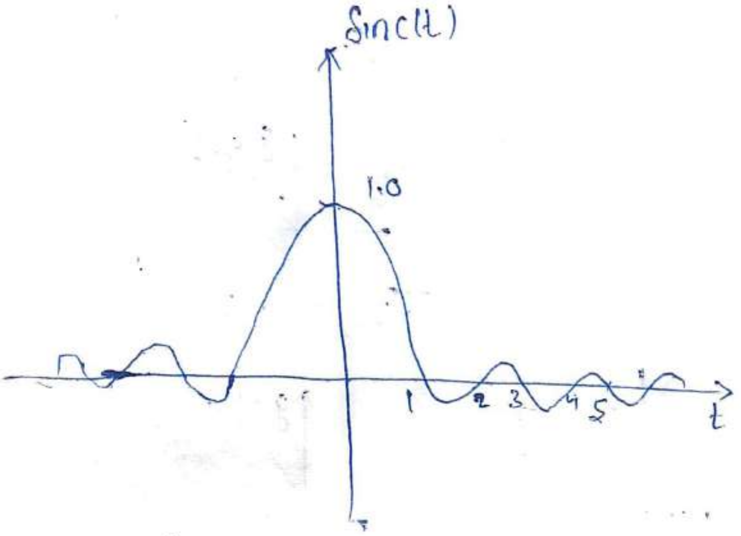
unit - Rectangular



$$\text{rect}(t) = \begin{cases} 1, & |t| < 1/2 \\ 1/2, & t = 0 \\ 0, & |t| > 1/2 \end{cases}$$

Sinc f'n

$$\text{Sinc}\, t = \frac{\sin \pi t}{\pi t}$$

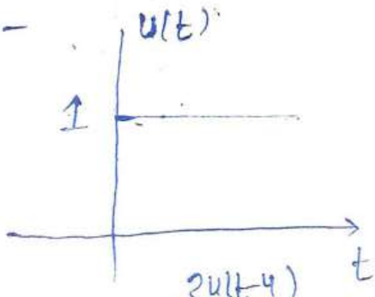


$\sin \pi t = 0$
$\pi t = \pi, 2\pi, 3\pi$
$1, 2, 3, 4 \ldots$

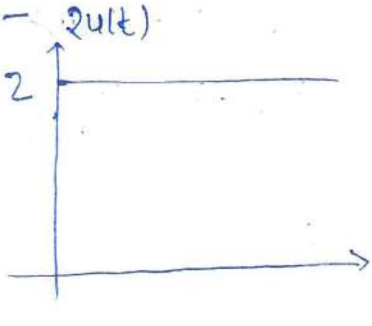
1. Amp. Scaling. $\longrightarrow g(t) \to Ag(t)$.
2. Time Scaling. $\longrightarrow g(t) \to g(t/2), g(2t), g(t/3), g(3t)$
3. Time shifting. $\longrightarrow g(t) \to g(t-t_0), g(t+t_0)$

Q. $u(t) \Rightarrow 2u(t-4)$

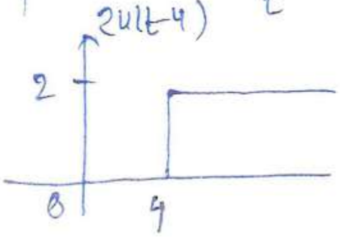
Step 1:-



Step 2:-

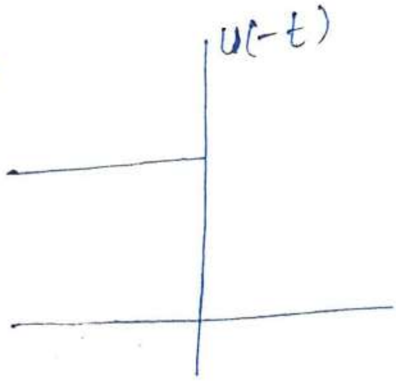


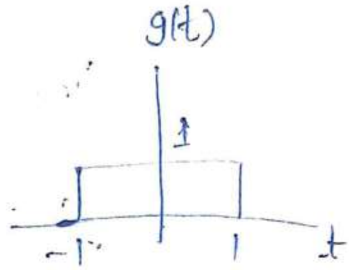
Step 3:-



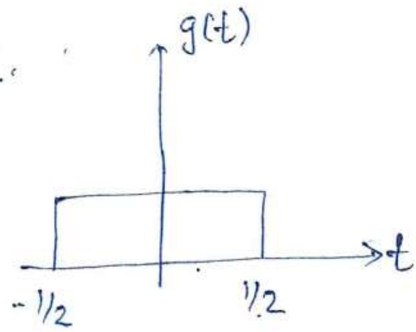
$$x(t) \rightarrow \left.\begin{array}{l} x(t-t_0) \; - \text{RHS.} \\ x(t+t_0) \; - \text{LHS} \end{array}\right\} (t_0 > 0)$$

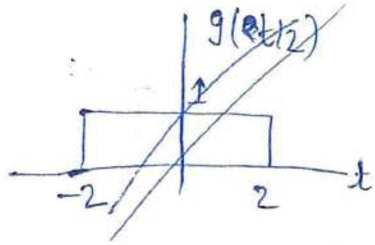
U(-t)



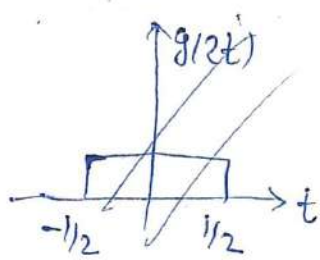


Q.1.





$$\rightarrow -2g\left(\frac{t+2}{4}\right)$$



Sol.ⁿ

$$\# \; g(t) \rightarrow A\,g\left(\frac{t-t_0}{a}\right)$$

$\downarrow$

1. Amp. Scaling
2. Time Scaling
3. Time shifting.

$$g(t) \rightarrow A\,g(t)$$
$$\downarrow t \rightarrow t/a$$
$$A\,g(t/a)$$
$$\downarrow t \rightarrow t-t_0$$
$$A\,g\left(\frac{t-t_0}{a}\right)$$

$\# \; A\,g(bt-t_0)$

$\downarrow$

1) Amp. scaling.
2) Time shifting
3) Time scaling.

$$g(t) \longrightarrow Ag(t)$$

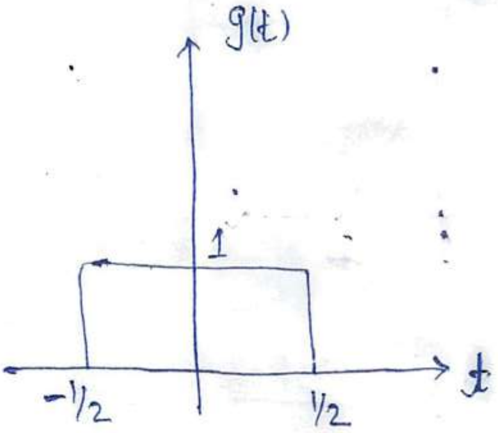
$$\Big\downarrow t \to t - t_0$$

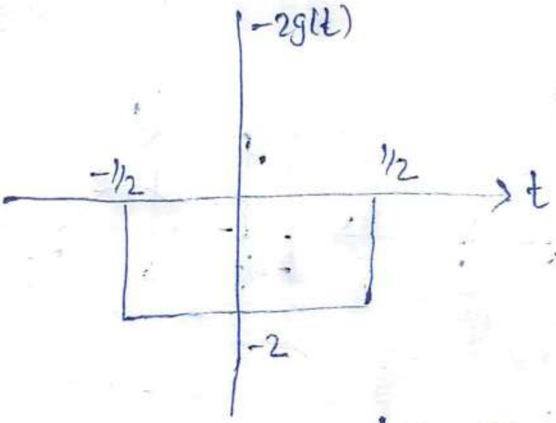
$$Ag(t - t_0)$$

$$\Big\downarrow t \to bt$$

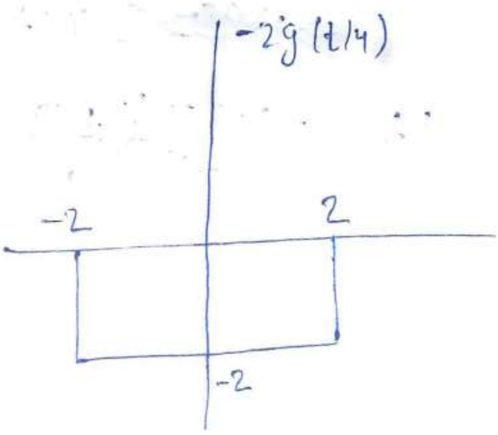
$$Ag(bt - t_0)$$

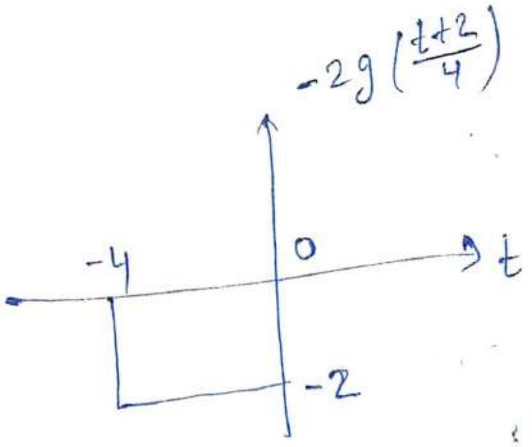
Ans 1.



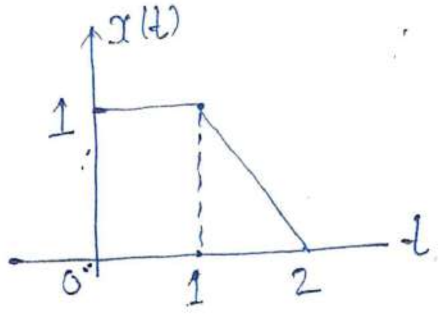


$$\Big\downarrow t \to t/4$$



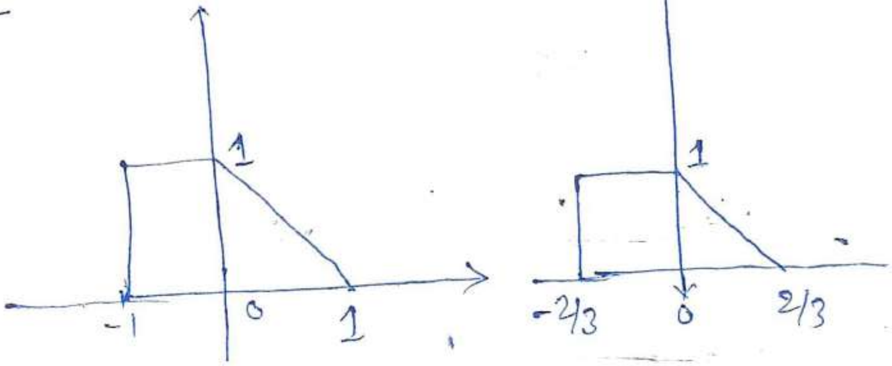


Q.

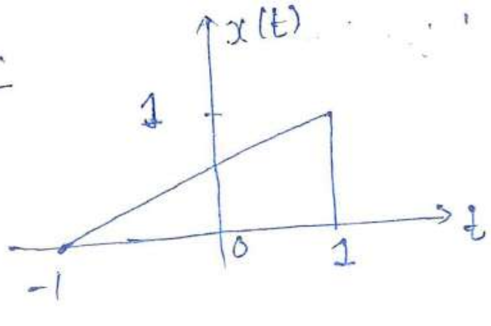


find $x\left(\frac{3}{2}t+1\right)$

Ans.



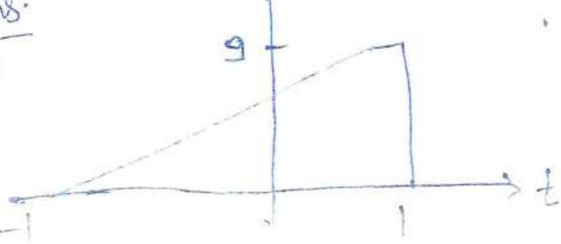
Q.

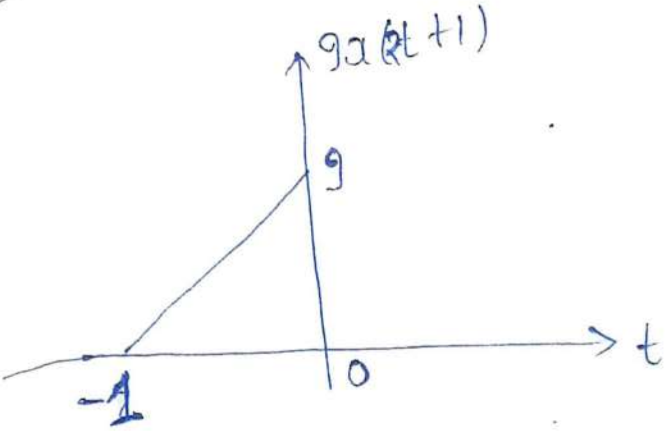


→ i) $9x(2t+1)$

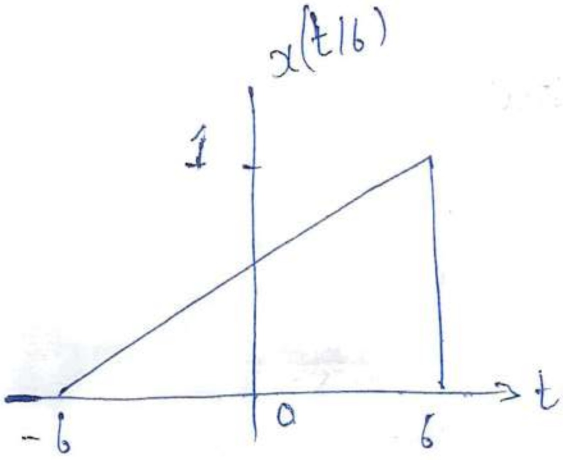
→ ii) $x\left(\frac{t+3}{6}\right)$

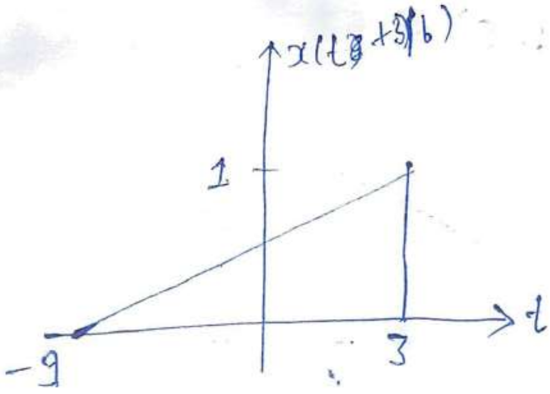
Ans.
(i)



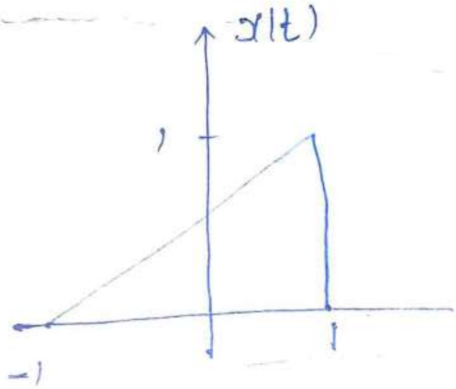


(II)



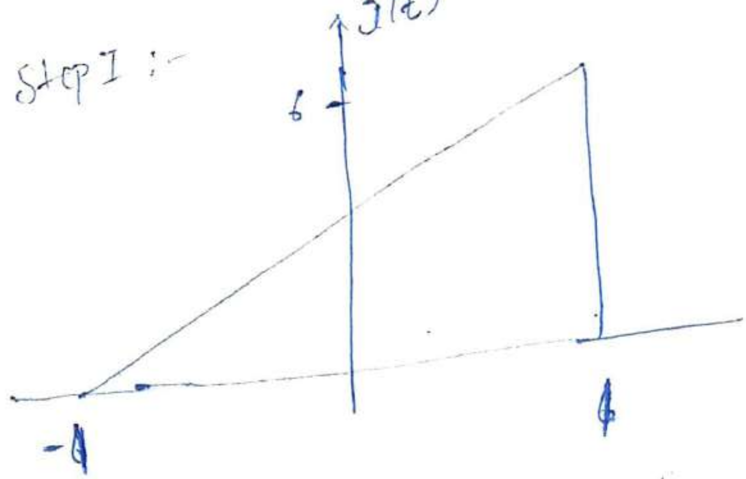


Q.

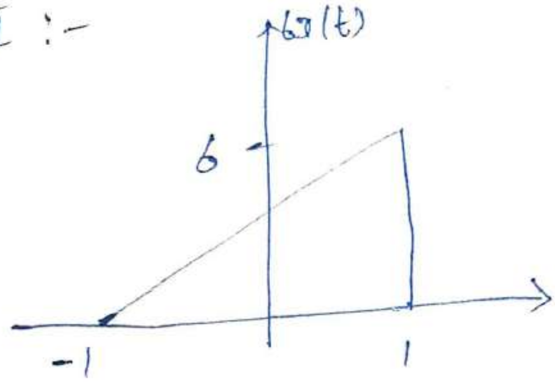


$\rightarrow 6x(-2t-5)$

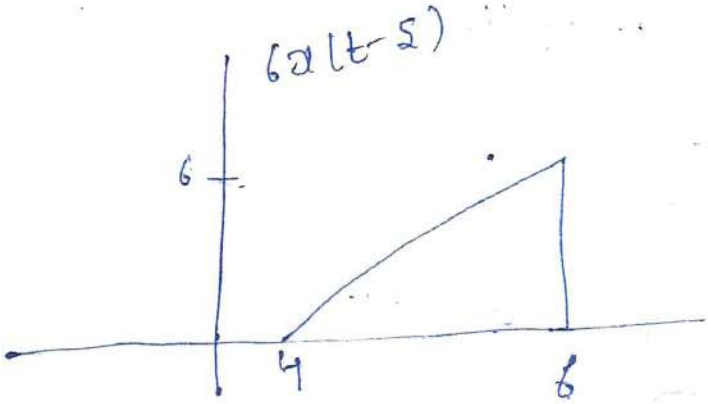
Step I :-



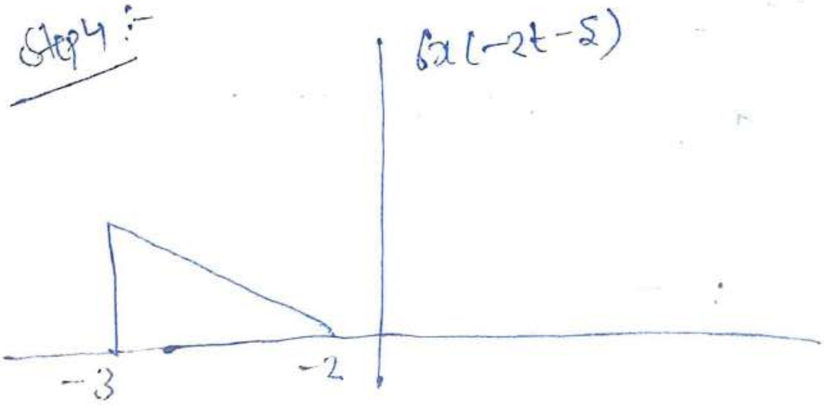
Step II :-



Step 3



Step 4 :-



Discrete Signal:

1. $$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{elsewhere} \end{cases}$$

$$r(n) = \begin{cases} n, & n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

$$\delta[an] = \delta[n]$$

But $\delta(at) = \frac{1}{|a|}\delta(t)$

2. $$\delta(t) = \frac{du(t)}{dt}$$

$$\delta[n] = u[n] - u[n-1]$$

3. $$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

Integration (t) = Summation in discrete

difference (t) = difference in discrete.

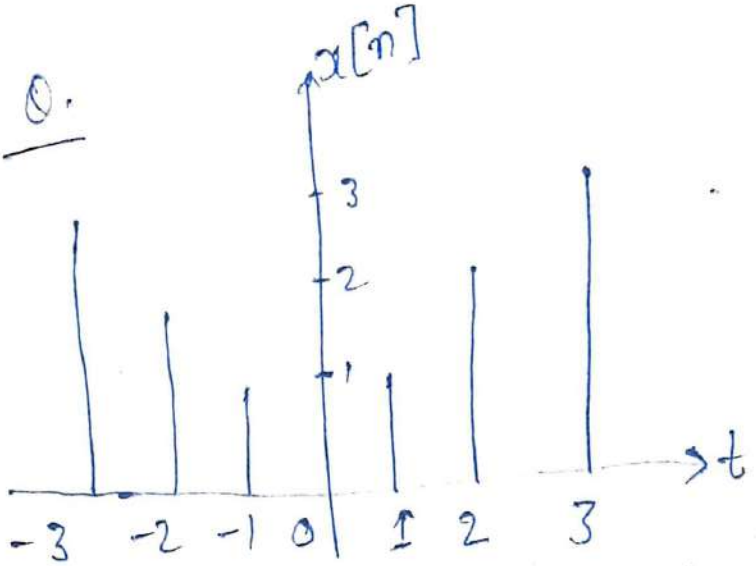
4. $$r(t) = t\,u(t)$$

$$r[n] = \sum_{m=-\infty}^{n-1} u[m]$$

5) $$u(t) = \frac{d}{dt} r(t)$$

$$u[n] = r[n+1] - r[n]$$

Q.



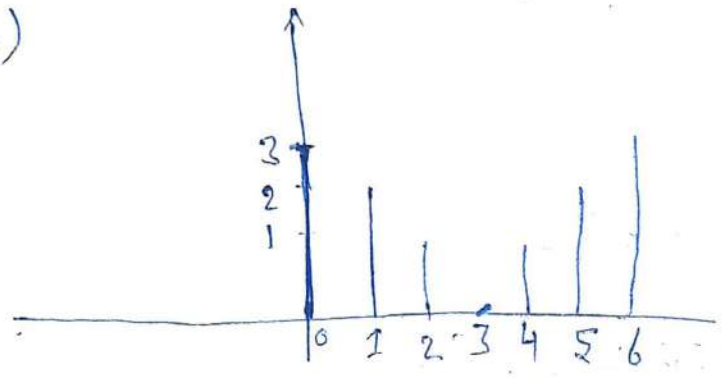
1) $x[n-3]$

2) $x(n+3)$

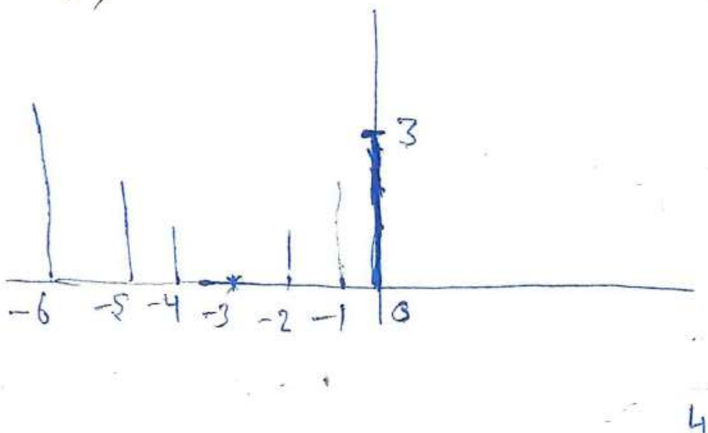
3) $x[n/2]$

4) $x[2n]$

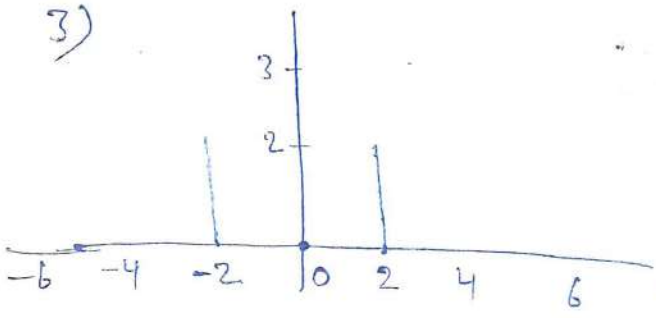
Ans. 1.)



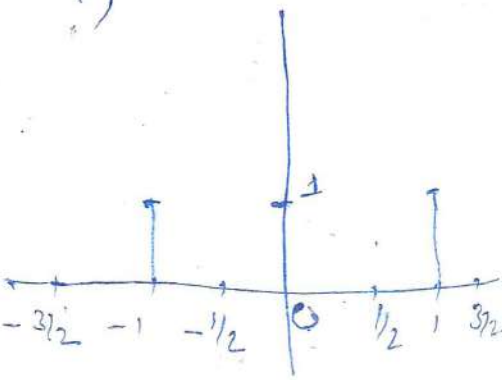
2)



3)



4)



Q. $x[n]$ is defined as

GATE-94

$$x[n] = \begin{cases} 0, & n<-2 \ \& \ n>4 \\ 1, & \text{otherwise} \end{cases}$$

Det. value of n, for which $x[-n-2]$ is guaranteed to be zero.

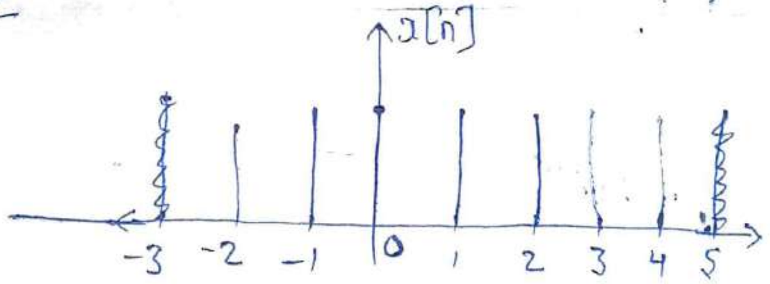
A) $n<1$ & $n>7$

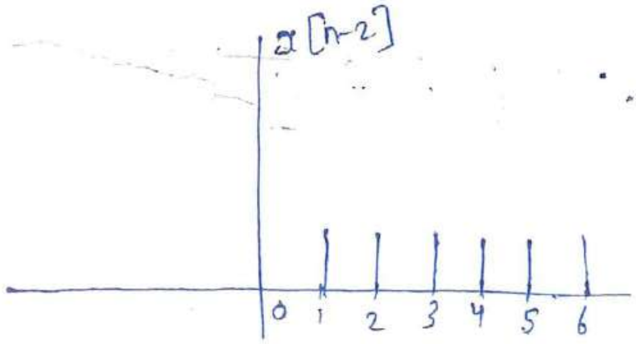
b) $n<-4$ & $n>2$

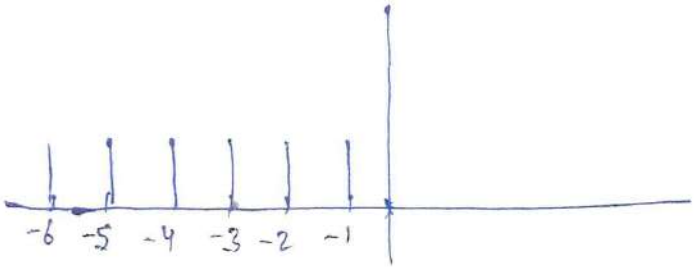
c) $n<-6$ & $n>0$ ✓

d) $n<-2$ & $n>4$.

Ans.







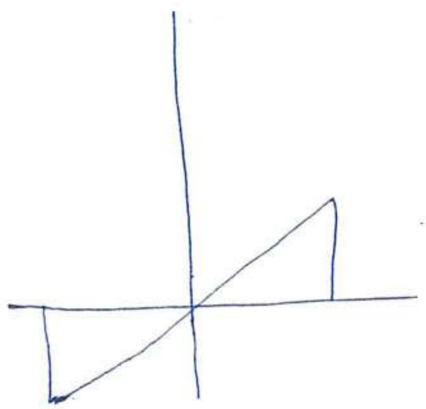
Even and odd part of signal:-

$x(-t) = x(t) \rightarrow$ Even f'n)

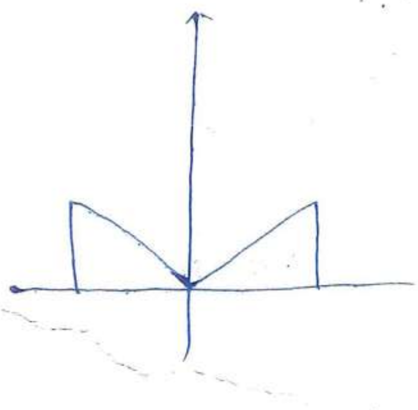
$x(-t) = -x(t) \rightarrow$ odd f'n.

[Note:- in case of odd function

$x(0) = 0$]. Not necessary only requirement



→ odd function



→ Even f^n.

$x(t) + x(-t) = 2x(t)$, for Even

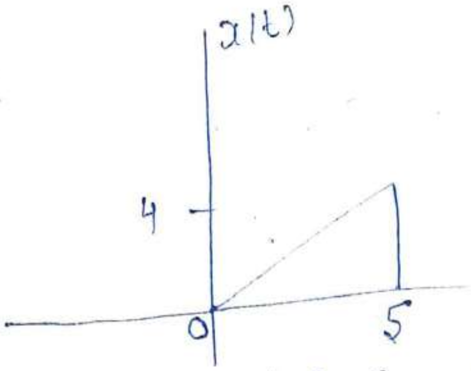
$x(t) - x(-t) = 0$, for odd

$f(t)$ is given

$$f_e(t) = \frac{f(t) + f(-t)}{2}$$

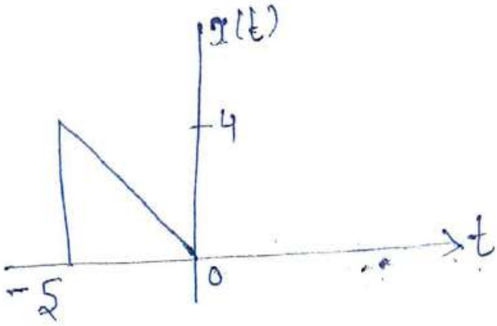
$$f_o(t) = \frac{f(t) - f(-t)}{2}$$

Q.



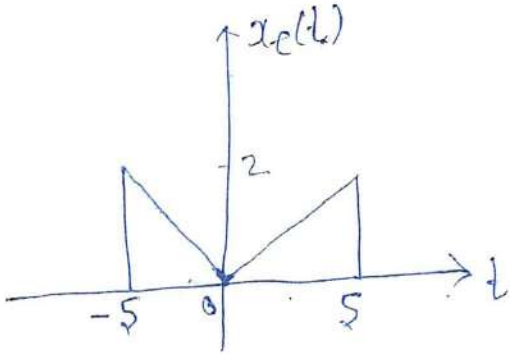
Draw odd & Even part of above graph.

Ans.



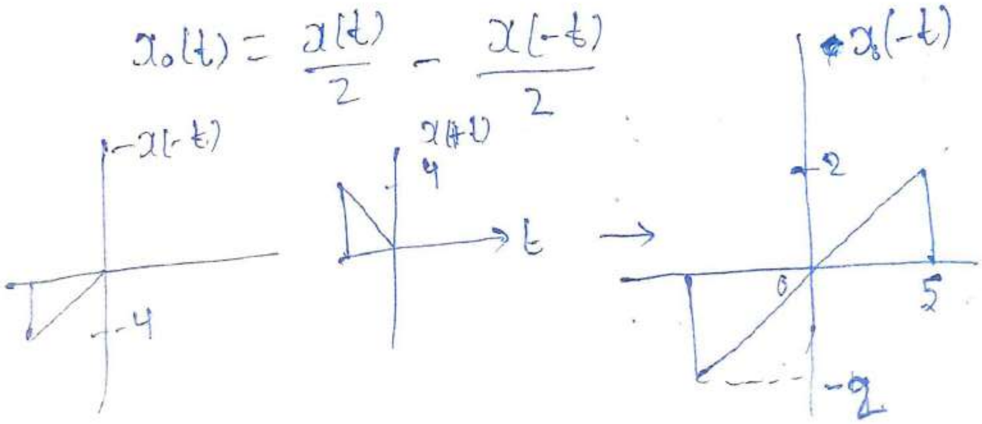
$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$= \frac{x(t)}{2} + \frac{x(-t)}{2}$$



Even part

$$x_o(t) = \frac{x(t)}{2} - \frac{x(-t)}{2}$$



odd part

Imp. charac. of Even & odd f'n :-

1) $\int_{-a}^{a} x(t)\,dt = 0$ if $x(t)$ is odd

2) $\int_{-a}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)\,dt$, if $x(t)$ is Even.

3.) $\sum_{K=-N}^{N} x[K] = x[0] + 2\sum_{K=1}^{N} x[K]$

if $x[K]$ is Even.

4) $\sum_{K=-N}^{N} x[K] = 0$, if $x[K]$ is odd.

Note :-

1) Sum & product of two Even f'n is always Even.
2) Sum of two odd f'n is always odd
3) product of two odd f'n) is alway Even
4) difference & Quotient of two Even is alway Even.
5) difference of two odd is alway odd.
6) quotient of two odd is always Even
7) Differentiation & Integration change odd to Even & Even to odd.
8) $\int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} x_e^2(t)\,dt + \int_{-\infty}^{\infty} x_o^2(t)\,dt$ 17

$x_e(t)$ & $x_o(t)$ are Even & odd parts respectively.

Q. $\int_{-1/10}^{1/10} t \sin(10\pi t)\, dt$ has zero value

$t \to$ odd, $\sin(10\pi t) \to$ odd $\to$ Even

1) True

2) False (✓)

$$= 2\int_0^{1/10} t \sin(10\pi t)\, dt$$

Q. $\int_{-1/20}^{1/20} 4t \cos(10\pi t)\, dt$ has zero value

$4t \to$ odd, $\cos(10\pi t) \to$ Even $\to$ odd

1) True (✓)

2) False

Q. odd & Even part of sinc f'n are

2 Marks

A) 0, Sinct (✓)

B) Sinct, 0

C) $\frac{1}{2}$ Sinct, 0

d) 0, $\frac{1}{2}$ Sinct

$$f(t) = \frac{\sin \pi t}{\pi t}$$

$$f(-t) = \frac{\sin(-\pi t)}{(-\pi t)} = \left(\frac{\sin \pi t}{\pi t}\right)$$

Q. Det. Even & odd part of

1). $u(t)$

2). $\delta(t)$

3. $\sin\left(\omega t + \frac{\pi}{4}\right)$

4. $\delta(n)$

5. $\mathrm{Sgn}(t)$

Sol.ⁿ

1) $\mathrm{Sgn}(t) = 2u(t) - 1$

$$u(t) = \frac{1 + \mathrm{Sgn}(t)}{2}$$

$$u(t) = \underbrace{\frac{1}{2}}_{\text{Even part}} + \frac{1}{2}\,\mathrm{Sign}(t)$$ ← odd part

= odd f'n.

2) $\mathrm{Sgn}(t) =$

$f_e = 0$

$f_o = \mathrm{Sgn}(t)$

3) $\delta(t) \rightarrow$ Even f'n.

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

$$\delta(-t) = \frac{1}{|-1|}\delta(t)$$

$$\delta(-t) = \delta(t)$$

$f_e(t) = \delta(t)$

$f_o(t) = 0$

4) $\delta(n)$ — Even

$\delta(an) = \delta(n)$

$\delta(-n) = \delta(n)$

$f_e(t) = \delta(n)$

$f_o(t) = 0$

17

Calculation of time period for discrete time system →

$x(n) = e^{j\omega_0 n}$ $\qquad x(t) = e^{j\omega_0 t}$

$x[n+N] = e^{j\omega_0 (n+N)}$

Let N is time period for discrete time system

$e^{j\omega_0 N} = 1$

$e^{j\omega_0 N} = e^{j2\pi}$ $\qquad x[n] = x[n+N]$ by property.

$\omega_0 = \frac{2\pi}{N}$ $\therefore$ $\boxed{N = \frac{2\pi}{\omega_0}}$

Difference b/w $x(t) = e^{j\omega_0 t} \rightarrow T_0 = \frac{2\pi}{\omega_0} \rightarrow e^{j\omega_0 t}$ is always Periodic for any value of T_0

$x[n] = e^{j\omega_0 n} \rightarrow N_0 = \frac{2\pi}{\omega_0}$

$T_0 = \frac{2\pi}{\omega_0} \rightarrow e^{j\omega_0 t}$ is always periodic for any value of T_0. This T_0 may be rational or irrational.

$e^{j\omega_0 n}$ is periodic for certain value of N_0.

→ N_0 must be always a rational No.

→ If N_0 is irrational no. then $e^{j\omega_0 n}$ is non-periodic

Ex. $x(t) = e^{3jt} \rightarrow \omega_0 = 3$, $T_0 = \frac{2\pi}{3} \rightarrow$ periodic

$x[n] = e^{3jn} \rightarrow \omega_0 = 3$, $N_0 = \frac{2\pi}{3} \rightarrow$ Non-periodic

Q. $x(t) = \cos\left(\frac{2\pi t}{12}\right)$

$\omega_0 = \pi/6$

$T_0 = 12$

$x[n] = \cos\left(\frac{2\pi n}{12}\right)$

$\omega_0 = \pi/6$

$N_0 = 12$ rational No.

Periodic.

Q. $x(t) = \cos \frac{8\pi t}{31}$ $\qquad x[n] = \cos \frac{8\pi n}{31}$

$\omega_0 = 8\pi/31$ $\qquad N_0 = (31/4) \approx 31$

$T_0 = 31/4$ $\qquad N_0 = 31$ Min. integer value.

Note:- In discrete time domain period N_0 is always integer value. But in continuous time there is no restriction for T_0.

Q. $x[n] = e^{j(3\pi/4)n}$

$\omega_0 = \frac{3\pi}{4}$

$N_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{3\pi/4} = 8/3$

$N_0 = 8$

Q. $x[n] = e^{j2\pi n/5}$

$N_0 = 5$

Q. $x[n] = e^{j(3n/4)}$

$\omega_0 = 3/4$

6/2 $\quad N_0 = \frac{2\pi}{\omega_0} = \frac{8\pi}{3}$ Not a rational no. therefore Non-periodic.

Q. $x[n] = e^{j(2\pi/3)n} + e^{j(\frac{3\pi}{4})n}$

$\downarrow \qquad\qquad \downarrow$

$N_0 = 3 \qquad N_0 = \frac{8}{3}$

$\downarrow \qquad\qquad \downarrow$

$\frac{3}{1} \qquad\qquad \frac{8}{1}$ → LCM, — HCF

$N_0 = 24$

Q. $x(t) = e^{j\frac{2\pi}{3}t} + e^{j\,3\pi/4\, t}$

$$T_0 = \frac{3}{1} \qquad T_0 = 8/3$$

$$T_0 = \frac{24}{1} = 24$$

Q. (i) $x(t) = 2\cos\left(\frac{9n\pi}{4}\right) - 3\sin\left(\frac{6n\pi}{5}\right)$

(ii) $x(t) = 2\cos\left(\frac{9\pi t}{4}\right) - 3\sin\left(\frac{6\pi t}{5}\right)$

Ans 1 (i) $\omega_{0_1} = \frac{9\pi}{4}$ $\qquad \omega_{0_2} = \frac{6\pi}{5}$

$N_{0_1} = \frac{8}{9} \qquad N_{0_2} = 5/3$

$\downarrow \qquad \downarrow$

$\frac{8}{1} \qquad \frac{5}{1}$

$N_0 = 40$

(ii) $T_{0_1} = \frac{8}{9} \qquad T_{0_2} = \frac{5}{3}$

$$\boxed{T_0 = \left(\frac{40}{3}\right)}$$

Q. 1) $x(t) = \cos 50\pi t + \sin 15\pi t$

11) $x[n] = \cos 50\pi n + \sin 15\pi n$

Ans 1. 1) $\omega_{0_1} = \frac{2\pi}{50\pi} = \frac{1}{25}$, $\omega_{0_2} = \frac{2}{15}$

$$T_0 = \frac{2}{5}$$

11) $\omega_{0_1} = \frac{2}{50}$, $\omega_{0_2} = \frac{2}{15}$

$\downarrow 2/1 \qquad \downarrow 2/1 \rightarrow N_0 = 2$

5) $x[n] = \cos(n/4)$

$$N_0 = \frac{2\pi}{1/4} = 8\pi$$

6) $x(n) = \cos^2(n\pi/8) = \dfrac{1 + \cos(n\pi/4)}{2}$

$$\omega_0 = \frac{\pi}{4},\ N_0 = \frac{2\pi}{\pi/4} = 8$$

$$N_0 = 8$$

7) $x[n] = \cos\left(\frac{n\pi}{3}\right) + \sin\left(\frac{n\pi}{4}\right)$

$N_{0_1} = 6 \quad N_{0_2} = 8$

$N_0 = 24$

$\frac{2\pi}{\pi/4}$

$\frac{2\pi}{\pi/3}$

Types of System :-

1. System with memory & without memory.
   ↓
   (Memory less System)

A System is called memory less if o/p for each value of independent variable at a given time depends upon i/p at the same time only.

$y[n] = 2x[n]$ → Memory-less system.

$y[n] = 2x[n-3]$ → Memory System. because o/p not depend on present value but past value.

$y[n] = 2x[n+3]$ → It talks about future value.

⇒ Resistor is a memory less N/w while capacitors & inductor have memory.

$y[n] = x[-n]$ → Memory as well as gives future value also.

$y[3] = x[-3]$

1. Accumulator :-

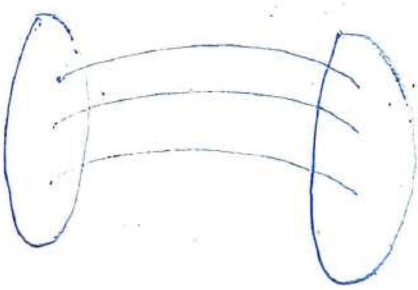
$$y[n] = \sum_{K=-\infty}^{n} x[K] = x[-\infty] + x[-\infty-1] + \dots\ x[-3] + x[-2] + x[-1] + x[0] + x[1] + \dots + x[n]$$

$$y[n] = y[n-1] + x[n]$$

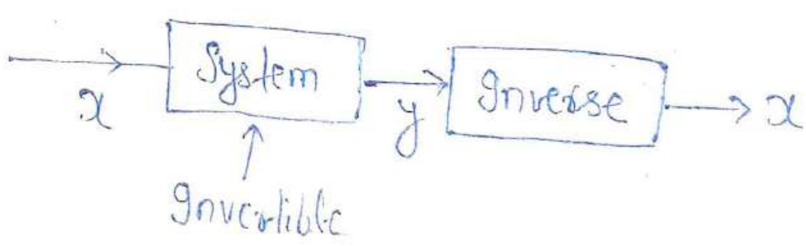
$\boxed{y[n] = y[n-1] + x[n]}$ → is the Equ. of accumulator

2. Invertibility :-

A System is said to be invertible if distinct i/p lead to distinct o/p. i.e one to one mapping Exist.



If a System is invertible then inverse will also Exist.



So, a system is called invertible if we can determine its i/p signal 'x' uniquely by observing its o/p signal 'y'.

Q. $y[n] = 0$, → Non invertible

Q. $y[n] = 2x[n]$ → Invertible

Q. $y[n] = [x[n]]^2$ → Non invertible

Q. Accumulator is

GATE I.

$y[n] = y[n-1] + x[n]$ invertible system

Q. What is the inverse of Accumulator.

IES-04

A) $y[n] = x[n] + x[n-1]$

B) $y[n] = x[n] - x[n-1]$ (✓)

c) $y[n] = x[n-1]$

d) $y[n] = y[n-1] + x[n]$

Note:- For calculation of inverse function simply put x on y & y on x.

$$y[n] = y[n-1] + x[n]$$

$$\downarrow$$

$$x[n] = x[n-1] + y[n]$$

$$y[n] = x[n] - x[n-1]$$

Causal System :- If o/p at any time depend upon value of i/p pres at present & past only i.e o/p does not depend upon future value of i/p then systems are called as causal system.

Q. which statement are true ?

I.E.S.

1. All system with memory are causal.

2. All memory less system are causal. ✓

A) 1, B) 2 (✓)

3) 1,2 D) None

2) Memory-less system means o/p depend on present i/p so, it is causal system.

1) A system which has memory sometime may depend on future value also.

$y[n] = x[-n]$

$y[3] = x[-3]$

$y[-3] = x[+3]$

It is Ex. of Non-causal system.

1. $y[n] = x[n+3]$ → Non-causal

2. $y[n] = x[-n]$ → N.C.

3. $y[n] = x[n^2]$ → Non-causal

4. $y[n] = [x[n]]^2$ → causal.