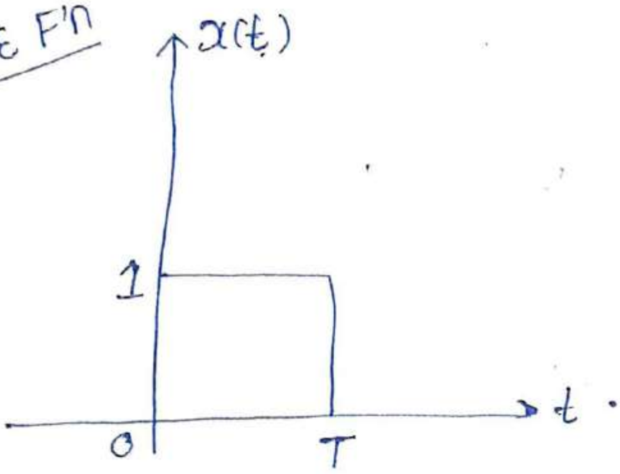
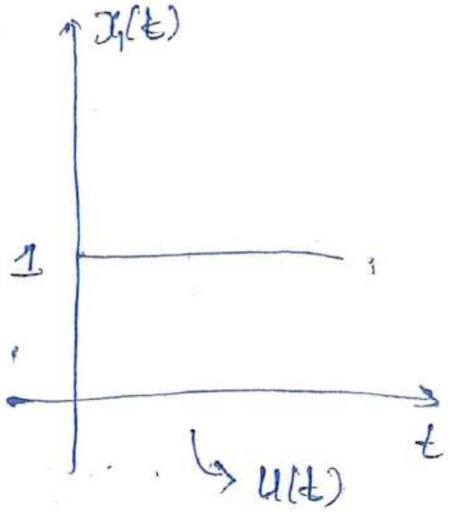
19/10/07

Laplace Transform

GATE F'n



$x(t) = u(t) - u(t-T)$



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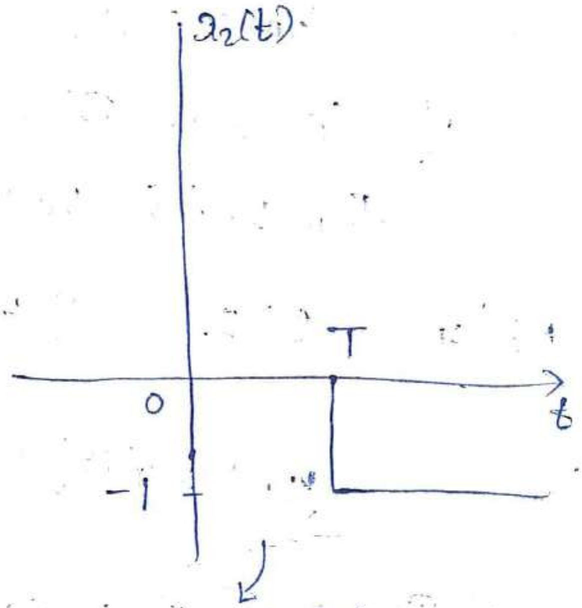
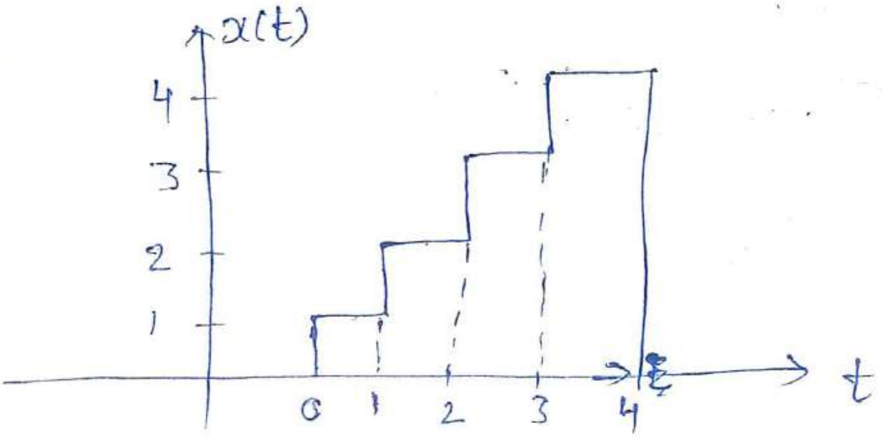


$-u(t-T)$

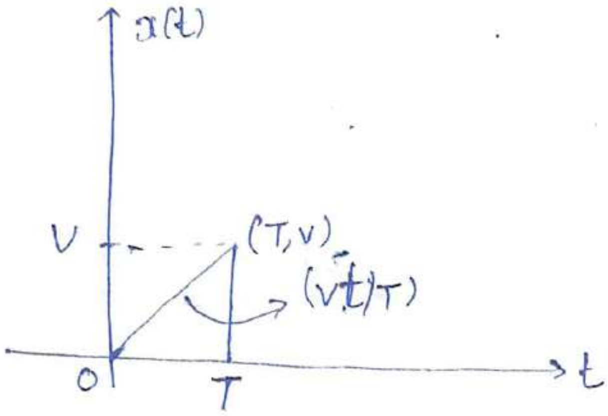
Q.



$$x(t) = 1\left[u(t) - u(t-1)\right] + 2\left[u(t-1) - u(t-2)\right]$$
$$+ 3\left[u(t-2) - u(t-3)\right] + 4\left[u(t-3) - u(t-4)\right]$$

$x(t) = u(t) + u(t-1) + u(t-2) + u(t-3) - 4u(t-4).$

Q. write the L.T of this fig.

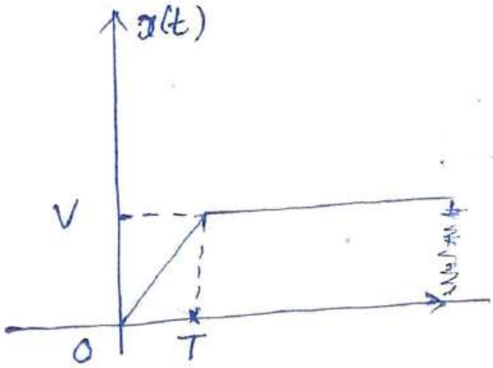


$$x(t) = \frac{Vt}{T}\left(u(t) - u(t-T)\right)$$

$$= \frac{Vt}{T}u(t) - \frac{V}{T}(t-T+T)\,u(t-T)$$

$$x(t) = \frac{Vt}{T}u(t) - \frac{V}{T}(t-T)\,u(t-T) - Vu(t-T).$$

Q.



$$x(t) = \frac{Vt}{T}\left(u(t) - u(t-T)\right) + Vu(t-T)$$

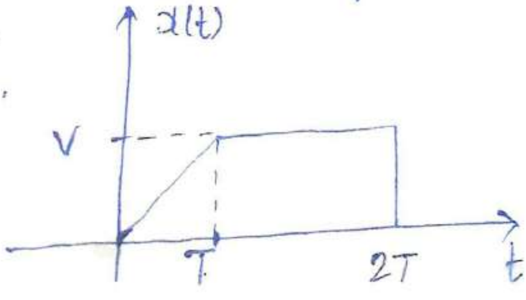
$$= \frac{Vt}{T}u(t) - \frac{V}{T}(t-T)\,u(t-T)$$

$$= \frac{Vt}{T}u(t) - \frac{V}{T}(t-T)\,u(t-T)$$

$$= \frac{V}{T}r(t) - \frac{V}{T}r(t-T)$$

$r(t) = t\,u(t)$

Q.



$$x(t) = \frac{Vt}{T}\left(u(t) - u(t-T)\right) + V\left(u(t-T) - u(t-2T)\right)$$

$$= \frac{Vt}{T}u(t) - \frac{V}{T}(t-T+T)u(t-T) + Vu(t-T)$$
$$- Vu(t-2T)$$

$$x(s) = \frac{V}{T}r(t) - \frac{V}{T}r(t-T) - Vu(t-2T)$$

$$X(s) = \frac{V}{Ts^2} - \frac{V}{Ts^2}e^{-sT} - \frac{Ve^{-2Ts}}{s}$$

$r(t) = t\,u(t)$
$R(s) = 1/s^2$

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt$$ — one sided Laplace Transform. used in Mathematics

$$F(s) = \int_{-\infty}^{\infty} f(t)e^{-st}\,dt$$ — Two sided Laplace Transform (concept of Roc is used)

1. $L(e^{-at}u(t)) = \frac{1}{s+a}$

2. $L\,e^{at}u(t) = \frac{1}{s-a}$

3. $L\,u(t) = 1/s$ (put $a=0$)

4. $L(\sin\omega_0 t) = \frac{\omega_0}{s^2+\omega_0^2}$

5. $L(\cos\omega_0 t) = \frac{s}{s^2+\omega_0^2}$

6. $L(\sinh\omega_0 t) = \frac{\omega_0}{s^2-\omega_0^2}$

7. $L(\cosh\omega_0 t) = \frac{s}{s^2-\omega_0^2}$

8) $\cos \omega_0 t = \dfrac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$

$\cosh \omega_0 t = \dfrac{e^{\omega_0 t} + e^{-\omega_0 t}}{2}$

9) $\sin \omega_0 t = \dfrac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$

$\sinh \omega_0 t = \dfrac{e^{\omega_0 t} - e^{-\omega_0 t}}{2}$

10) $L(t^n) = \dfrac{\lfloor n}{s^{n+1}}$

11) 1. $L\, f(t-t_0)\, u(t) = e^{-t_0 s} F(s)$ [wrong]

2. $L\, f(t)\, u(t-t_0) = e^{-t_0 s} F(s)$ wrong

3. $L\, f(t-t_0)\, u(t-t_0) = e^{-t_0 s} F(s)$ (✓)

Q. $x(t) \longrightarrow X(s)$

$t\,x(t) \longrightarrow -\dfrac{dX(s)}{ds}$

$L(t^n e^{-at}) = \dfrac{\lfloor n}{(s+a)^{n+1}}$

$L(t e^{-at}) = \dfrac{1}{(s+a)^2}$

$L\left(f(t)/t\right) = \int_s^{\infty} F(s)\,ds.$

Q. What is Laplace transform of $\dfrac{\sin \omega_0 t}{t}$?

GATE

a) $\tan^{-1}(\omega_0/s)$ b) $\tan^{-1}(s/\omega_0)$

c) $\dfrac{\omega_0}{s^2+\omega_0^2}$ d) Not defined (✓)

2/10/01

$$1+\gamma+\gamma^2+ \cdots +\gamma^n = \frac{1\cdot(1-\gamma^{n+1})}{(1-\gamma)}$$

$$\gamma^0+\gamma^1+\gamma^2+ \cdots +\gamma^n$$

$\cancel{\gamma+\gamma^2+}$

$$\left.\begin{array}{l} 1+4+16+64 \\ \quad \gamma=4,\ a=1 \\ 4^0+4^1+4^2+4^3 \end{array}\right\} = \frac{1(4^4-1)}{3} = \frac{255}{3} = 85$$

Q. why we need Laplace transform?

→ Laplace Transform is Extension of fourier transform.

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

This F.T. will exist only when

$$\int_{-\infty}^{\infty} |f(t)|\, dt < \infty$$

e.g. Ramp → (This condition not valid)

So, in case of Ramp we cannot calculate F.T. Similarly for parabola we cannot calculate fourier transform.

i.e Ramp and parabolic f'n are not converged

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \times e^{-\sigma t}$$ here σ must be +ve. $(\sigma > 0)$.

$$F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-(\sigma+j\omega)t}\, dt$$

where $s = \sigma + j\omega$

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

$\downarrow$

L.T of $f(t)$

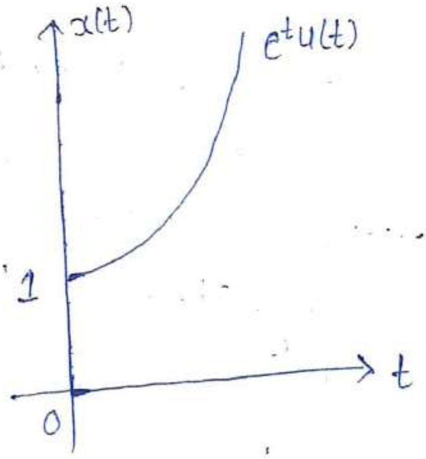
L.T. will be valid if

$$\int_{-\infty}^{\infty} |f(t)| e^{-\sigma t} dt < \infty$$

Q. For signal shown ~~above~~

GATE-2 Marks

IES-05



a) only F.T Exist

b) only L.T ,, (✓)

c) LT & FT Exist

d) Neither LT & FT Exist.

Ans.

$$FT = \int_{0}^{\infty} |e^t| dt \neq \text{finite}$$

$$LT = \int_{0}^{\infty} |e^t \times e^{-\sigma t}| dt < \infty$$

Calculation of L.T :-

Q. $$F(s) = \int_{0}^{\infty} f(t) e^{-st} dt \text{ — one sided L.T.}$$

$$= \int_{-\infty}^{\infty} f(t) e^{-st} dt \text{ — Two sided L.T.}$$

2 Q. $x(t) = e^{-at}u(t)$

Ans.

$$L.T = \int_{-\infty}^{\infty} |f(t)| e^{-st}\, dt$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st}\, dt$$

$$= \int_{0}^{\infty} e^{-at} e^{-(\sigma + j\omega)t}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+\sigma+j\omega)t}\, dt$$

$$X(s) = \frac{1}{a+\sigma+j\omega}, \quad Re\{s\} > -a$$

Calculation of Roc

$$\int_{0}^{\infty} e^{-(a+\sigma)t} e^{-j\omega t}\, dt$$

for Roc $(\sigma + a) > 0$

$$\Rightarrow \sigma > -a$$

σ = Real part of s $> -a$

$$Re\{s\} > -a$$

Q. $x(t) = -e^{-at}u(-t)$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st}\, dt$$

$$= -\int_{-\infty}^{0} e^{-at} e^{-(\sigma + j\omega)t}\, dt = -\int_{-\infty}^{0} e^{-(\sigma+a)t} e^{-j\omega t}\, dt$$

$$= -\int_{0}^{\infty} e^{-(a+\sigma+j\omega)t}\, dt$$

So for RoC $a+\sigma<0$

$$Re\{s\} < -a$$

$$X(s) = \frac{1}{a+\sigma+j\omega} \quad Re\{a\} < -a$$

1). $x(t) = e^{-at}u(t)$

$$= \frac{1}{a+s}, \quad Re\{s\} > -a$$

2) $x(t) = e^{-at}u(-t)$

$$= -\frac{1}{a+s}, \quad Re\{s\} < -a$$

Q. What is L.T. of $u(t)$

Ans.

$$x(t) = e^{-at}u(t)$$

$$= \frac{1}{s+a}, \quad Re\{s\} > -a$$

Put $a=0$

$$= \frac{1}{s}, \quad Re\{s\} > 0$$

Q. What is Laplace Transform of $u(-t)$

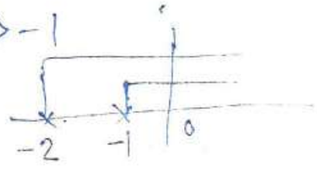
Ans. $X(s) = -\frac{1}{s} \quad Re\{s\} < 0$

Q. What is L.T of $x(t) = 3e^{-2t}u(t) - 2e^{-t}u(t)$

Ans.

$$X(s) = \frac{3}{s+2} - \frac{2}{s+1}$$

RoC $Re\{s\} > -2$, $Re\{s\} > -1$



over-all RoC will be $Re\{s\} > -1$

Q. $x(t) = e^{-2t}u(t) + e^{-3t}\cos 3t\, u(t)$

2 $= \frac{\cancel{1}}{s+2} + \cancel{\frac{s+3}{(s+3)^2+9}}$

$$e^{-3t}\cos 3t = \frac{e^{-3t}\left(e^{j3t} + e^{-j3t}\right)}{2}$$

$$= \frac{e^{-3t(1-j)} + e^{-3t(1+j)}}{2}$$

$$x(t) = e^{-2t}u(t) + \frac{e^{-3t(1-j)} + e^{-3t(1+j)}}{2}u(t)$$

$$X(s) = \frac{1}{s+2} + \left[\frac{1}{s+3(1-j)} + \frac{1}{s+3(1+j)}\right] \times \frac{1}{2}$$

$Re\{s\} > -2$ $\quad$ $Re\{s\} > -3$

overall
$Roc\{s\} > -2$

1.Note:- If $x(t)$ is of finite duration then Roc will be Entire s-plane.

* ① $x(t) = \begin{cases} e^{-at}, & 0 < t < T \\ 0, & \text{Elsewhere} \end{cases}$

Roc will be Entire s-plane.

* ② $\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & \text{Elsewhere} \end{cases}$

So, Roc of $\delta(t)$ will be Entire s-plane.

2. Note :- If $x(t)$ is Two Sided function then Roc will consist of a strip in - S-plane.

e.g. $x(t) = e^{-a|t|}$

$$= \left.\begin{matrix} e^{-at}, & t > 0 \\ e^{at}, & t < 0 \end{matrix}\right\}$$ It Roc will consist of a strip.

Q. what is Laplace transform of $x(t) = \delta(t) - \frac{4}{3} e^{-t}u(t) + \frac{1}{3} e^{2t}u(t)$.

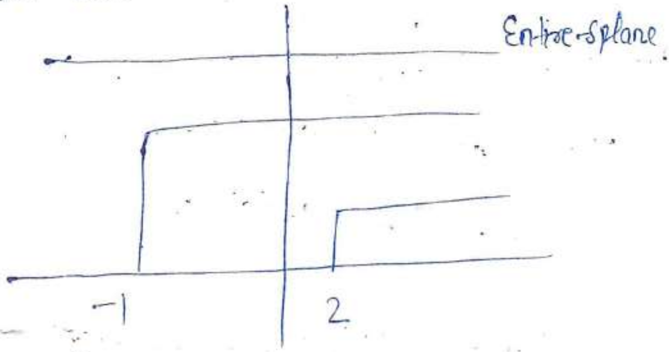
Ans.

$$X(S) = 1 - \frac{4}{3} \times \frac{1}{S+1} + \frac{1}{3} \frac{1}{(S+2)}$$

$$= 1 - \frac{4}{3(S+1)} + \frac{1}{3(S+2)}$$

Entire S-plane $\quad$ $Re\{s\} > -1$ $\quad$ $Re\{s\} > +2$

Overall Roc will be $Re\{s\} > 2$



Q. Calculate L.T. of

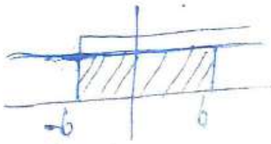
$$x(t) = e^{-b|t|}$$

Ans.

$$x(t) = e^{-bt} u(t) + e^{+bt}u(-t)$$

$$= \frac{1}{s+b} \cancel{+} \frac{1}{s-b}$$

$Re\{s\} > -b$ $\quad$ $Re\{s\} < b$



overall Roc will be $-b < Re\{s\} < b$

$$\frac{8s+10}{(s+1)(s+2)^3} = \frac{A}{s+1} + \frac{Bs+C}{(s+2)^2} + \frac{Ds+}{(s+2)^3}$$

2

properties of L.T :-

1. $x(t) \xrightarrow{L.T.} X(s)$ $\quad RoC = \{R\}$,
$x(t-t_0) \xrightarrow{L.T.} e^{-t_0 s} X(s)$ $\quad RoC\{R\}$

2. $x(t) \xrightarrow{L.T.} X(s)$ $\quad RoC = R$
$e^{s_0 t} x(t) \xrightarrow{L.T.} X(s-s_0)$ $\quad RoC = R + Re\{s_0\}$.

3. $x(t) \xrightarrow{L.T.} X(s)$ $\quad RoC = \{R\}$
$x(at) \xrightarrow{L.T.} \frac{1}{|a|} X(s/a)$ $\quad RoC = \{aR\}$

$x(-t) \xrightarrow{L.T.} X(-s)$ $\quad RoC = \{-R\}$.

4. $x(t) \xrightarrow{L.T} X(s)$ $\quad RoC = \{R\}$
$\frac{dx(t)}{dt} \xrightarrow{L.T.} sX(s)$ $\quad RoC = \{R\}$.

5. $x(t) \longrightarrow X(s)$ $\quad RoC = \{R\}$
$t x(t) \longrightarrow -\frac{dX(s)}{ds}$ $\quad RoC = R$

6. $x(t) \longrightarrow X(s)$ $\quad RoC = R$

$$\int_0^t x(\tau) d\tau = \frac{1}{s} X(s) \quad RoC = R \cap \{Re\{s\}\} > 0$$

Q. $X(s) = \frac{10 s^2}{(s+1)(s+3)}$

$x(t) = ??$

Q. If $x(t) = \sin(\omega t + \alpha)$
what is L.T ?

A) $\frac{\alpha}{s^2+\alpha^2} \exp(s/\alpha\omega)$

B) $\frac{\omega}{s^2+\omega^2} \exp(s/\alpha\omega)$ (✓)

c) $\frac{s}{s^2+\alpha^2} \exp(s/\alpha\omega)$

d) $\frac{\omega}{s^2+\omega^2} \exp(s/\alpha\omega)$

Ans.

$$x(t) = \sin\omega t \cos\alpha + \cos\omega t \sin\alpha$$

$$= \frac{\omega}{s^2+\omega^2}\cos\alpha + \frac{s}{s^2+\omega^2}\sin\alpha$$

$$= \frac{1}{s^2+\omega^2}(\omega\cos\alpha + s\sin\alpha)$$

$$= \frac{\omega}{s^2+\omega^2}\left(\cos\alpha + \frac{s}{\omega}\sin\alpha\right)$$

$$= \frac{\omega}{s^2+\omega^2}(\cos\alpha + j\sin\alpha)$$

$$= \frac{\omega}{s^2+\omega^2}\, e^{\left(\frac{\alpha s}{\omega}\right)}$$

OR put $\alpha = 0$,

Q. $F(s)$ Z

$$f(0) = \lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

Initial value theorem

$$f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$ → final value theorem.

$$= \frac{1}{4}\cdot\frac{1}{4\left[\left(s+\frac{1}{2}\right)^2+1\right]}$$

Note:- In case of $x(t)$ if values are interms of trignometric function then generally initial value concept is applied.

But if $x(t)$ is in terms of Exponential f'n then final value concept is applied.

23/10/07

Causality of a System —

If $h(t)=0$ for $t<0$, then System is called as causal System.

Eg. $h(t) = e^{-t}u(t)$

$$H(s) = \left(\frac{1}{s+1}\right) \text{ Roc :- } Re\{s\} > -1$$

If Roc is Right of Right most pole; then system will be causal for rational function.

Q. $h(t) = e^{-|t|}$

$h(t) = e^{-t}u(t) + e^{t}u(-t)$

~~$H(s) =$~~ for $t<0$ $h(t) = e^{+t}$ $\Rightarrow$ System is Non causal

$$H(s) = \frac{1}{s+1} - \frac{1}{s-1}$$

$$= \frac{-2}{s^2-1}$$

Roc :- $-1 < Re\{s\} < 1$

Since, Non-causal System.

Stability of a System →

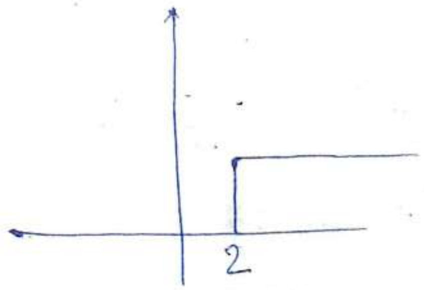
for stable System

in time domain $\int_{-\infty}^{\infty} |h(t)| dt < \infty$

In S-domain System will be stable, if RoC includes $j\omega$-axis.

Q. $H(S) = \dfrac{(S-1)}{(S+1)(S-2)}$, $Re\{s\} > 2$

Ans.



It is a causal System.
As RoC is Right of Right Most pole
Since RoC does not include $j\omega$-axis so unstable System.

Q. $h(t) = u(t)$

Ans. $\int_{-\infty}^{\infty} |u(t)| dt = \int_{0}^{\infty} 1 . dt = \infty$

→ So, it is unstable system

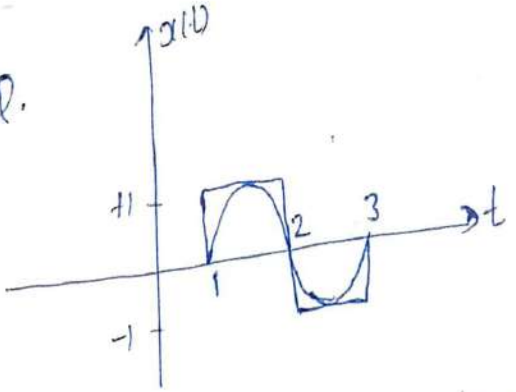
$H(s) = \dfrac{1}{S}$ $Re\{s\} > 0$

↳ It does not include $j\omega$-axis So, unstable System.

Q. If $h_1(t) = 1$
$h_2(t) = u(t)$
$h_3(t) = \dfrac{u(t)}{t+1}$
$h_4(t) = e^{-3t} u(t)$

which of the following is Causal & Stable?

Q.

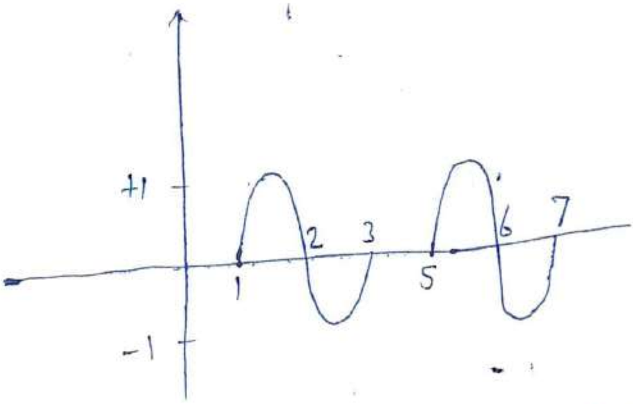


Ans. $x(t) = \sin\pi(t-1)\left[u(t-1) - u(t-2)\right]$

$-\sin\pi(t-2)\left[u(t-2) - u(t-3)\right]$

$= \sin\pi(t-1)\,u(t-1) - \sin\pi(t-3)\,u(t-3)$

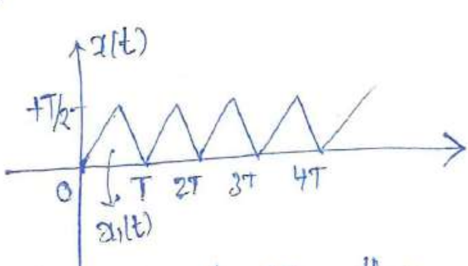
Q. $x(t)$?



$$x(t) = \sin\pi(t-1)\,u(t-1) - \sin\pi(t-3)u(t-3) - \sin\pi(t-7)u(t-7)$$
$$+ \sin\pi(t-5)\,u(t-5)$$

$$X(s) = \frac{\pi}{s^2+\pi^2}e^{-s} - \frac{\pi}{s^2+\pi^2}e^{-3s} + \frac{\pi}{s^2+\pi^2}e^{-5s} - \frac{\pi}{s^2+\pi^2}e^{-7s}$$

L.T. of a Periodic function →



If f'n has Period T then

$$X(s) = \frac{X_1(s)}{1-e^{-Ts}}$$

Z- Transform :-

$$X(Z) = \sum_{n=-\infty}^{\infty} x[n] Z^{-n}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$
↳ D.T.F.T.

If $Z = re^{j\omega}$

$$X(re^{j\omega}) = \sum_{n=-\infty}^{\infty} (r(e^{j\omega}))^{-n} x[n]$$

$$= \sum_{n=-\infty}^{\infty} (x[n].r^{-n}).e^{-j\omega n}$$

So, It is clear from Expression at

Z-transform is DTFT of $x[n]r^{-n}$.

⇒ If $r = 1$, then both Z-transform & DTFT are same.

Condition for Existence of L.T →

$$\int_{-\infty}^{\infty} |x(t)| e^{-\sigma t} dt < \infty.$$

Similarly, condition of Existence of Z-transform is

$$\sum_{n=-\infty}^{\infty} |x[n]r^{-n}| < \infty$$

Q. IES-06 Which one of the following is correct statement? The Region of Convergence of Z-transform of $x[n]$ consist of value of Z, for which $x[n] r^{-n}$ is

A) Absolutely Integrable

B) " Summable (✓)

c) unity

d) < 1

Q. What is Z-transform of $x[n] = a^n u[n]$

Ans.

$$X(Z) = \sum_{n=-\infty}^{\infty} x[n] Z^{-n}$$

$$= \sum_{n=0}^{\infty} (a^n Z^{-n})$$

$$= \sum_{n=0}^{\infty} (aZ^{-1})^n = 1 + aZ^{-1} + a^2Z^{-2} + \cdots$$

$$= \frac{1}{1-aZ^{-1}}, \text{ Roc} : |Z| > |a|$$

for Roc: $|aZ^{-1}| < 1$

$\Rightarrow |Z| > |a|$

Q. What is Z-transform of $x[n] = u[n]$

Ans. put $a = 1$

$$X(Z) = \frac{1}{1-Z^{-1}}, \text{ Roc: } |Z| > 1$$

Q.

$$\left.\begin{aligned} e^{-at}u(t) &\rightarrow \frac{1}{S+a} \quad Re\{s\} > -a \\ -e^{-at}u(-t) &\longrightarrow -\frac{1}{S+a} \quad Re\{s\} < -a \end{aligned}\right\} \text{In Laplace Transform.}$$

Q. $x[n] = -a^n u(-n-1)$

$$\left\{\begin{aligned} -n-1 &> 0 \\ -n &> 1 \\ n &< -1 \end{aligned}\right\}$$

$$X(Z) = \sum_{n=-\infty}^{\infty} x[n] Z^{-n}$$

$$= \sum_{n=-\infty}^{-1} -a^n Z^{-n}$$

$$= \sum_{n=1}^{\infty} -a^{-n} Z^n$$

$$= 1 - \sum_{n=0}^{\infty} (a^{-1}Z)^n$$

$$= \frac{1}{1-aZ^{-1}}, \text{ Roc: } |a^{-1}Z| < 1$$

$$|Z| < |a|$$

Note:

$$x[n] = a^n u(n)$$

$$= \frac{1}{1 - aZ^{-1}}, \text{ RoC } |Z| > |a|.$$

$$x[n] = -a^n u(-n-1)$$

$$= \frac{1}{1 - aZ^{-1}}, \text{ RoC } |Z| < |a|.$$

Q. (2004) If Z-transform of a system is $H(Z) = \frac{Z}{Z - 0.2}$. If RoC is $|Z| < 0.2$ then what is Impulse Response of a system

A) $(0.2)^n u[n]$ B) $(0.2)^n u(-n-1)$

C) $-(0.2)^n u(n]$ D) $-(0.2)^n u[-n-1]$ ($\checkmark$)

Ans:

$$H(Z) = \frac{1}{1 - 0.2Z^{-1}}, \quad |Z| < 0.2$$

Q. $x[n] = 7(1/3)^n u[n] - 6(1/2)^n u[n]$

$$X(Z) = \frac{7}{1 - 1/3 Z^{-1}} - \frac{6}{1 - \frac{1}{2} Z^{-1}}$$

For RoC:

$|\frac{1}{3} Z^{-1}| < 1$ $\qquad \frac{1}{2} Z^{-1} < 1$

$|Z| > \frac{1}{3}$ $\qquad \Rightarrow |Z| > \frac{1}{2}$

Overall RoC $|Z| > \frac{1}{2}$

Q. $x[n] = 7(\frac{1}{3})^n u(n) + 6(\frac{1}{2})^n u(-n-1)$

$$= \frac{7}{1 - \frac{1}{3} Z^{-1}} - \frac{6}{1 - \frac{1}{2} Z^{-1}}$$

$$\left|\frac{1}{3}z^{-1}\right| < 1 \qquad \left|\frac{1}{2}z^{-1}\right| > 1$$

$$|z| > \frac{1}{3} \qquad |z| < \frac{1}{2}$$

overall ROC: $\frac{1}{3} < |z| < \frac{1}{2}$.

Q. $x(n) = 7\left(\frac{1}{3}\right)^n u[n-1] + 6\left(\frac{1}{2}\right)^n u(-n-1)$

$$= \frac{-7}{1-\frac{1}{3}z^{-1}} - \frac{6}{1-\frac{1}{2}z^{-1}}$$

$$= \left|\frac{1}{3}z^{-1}\right| > 1 \qquad \left|\frac{1}{2}z^{-1}\right| > 1$$

$$|z| < \frac{1}{3} \qquad |z| \cancel{>} < \frac{1}{2}$$

overall ROC: $|z| < 1/3$

Q. $x[n] = 7\left(\frac{1}{3}\right)^n u[-n-1] + 6\left(\frac{1}{2}\right)^n u[n]$

$$= \frac{-7}{1-\frac{1}{3}z^{-1}} + \frac{6}{1-\frac{1}{2}z^{-1}}$$

ROC: $|z| < \frac{1}{3}$ $\qquad |z| > \frac{1}{2}$

there is no common portion for ROC & hence no Z-transform will exist for it.

Q. $x[n] = \left(\frac{1}{3}\right)^n \sin\left(\frac{n\pi}{4}\right) u[n]$

Ans.

$$\left(\frac{1}{3}\right)^n \left[\frac{e^{jn\pi/4} - e^{-jn\pi/4}}{2j}\right] u[n]$$

$$= \frac{1}{2j}\left[\left(1/3\, e^{j\pi/4}\right)^n - \left(1/3\, e^{-j\pi/4}\right)^n\right] u[n]$$

$$= \frac{1}{2j}\left[\frac{1}{1-\frac{1}{3}e^{j\pi/4}z^{-1}} - \frac{1}{1-\frac{1}{3}e^{-j\pi/4}z^{-1}}\right]$$

$$\left|\frac{1}{3}e^{j\pi/4}z^{-1}\right| < 1 \qquad |z| > \frac{1}{3}$$

property:-

$$x[n] \xrightarrow{Z.T.} X[Z] \qquad \gamma_1 < ROC < \gamma_2$$

$$a^n x[n] \xrightarrow{Z.T.} X[Z/a], \; a\gamma_1 < ROC < a\gamma_2$$

Q. If ~~x[n] =~~ $x(Z) = \cancel{\dfrac{Z\cdot e^{j}}{Ze^{-j} - a}} \cdot \dfrac{Z}{Z - ae^{j}}$ — ①

what is. $x[n]$, $Roc: |Z| > |a|$

Ans.

$$a^n u[n] \longrightarrow \frac{1}{1 - aZ^{-1}} \quad |Z| > |a|$$

$$\text{from ①} \rightarrow \frac{Z/Z}{1 - ae^{j}Z^{-1}} = \frac{1}{1 - ae^{j}Z^{-1}}$$

$$e^{jn} a^n u[n] \longrightarrow \frac{Z/e^{j}}{Z/e^{j} - a} = \frac{Z}{Z - ae^{j}}$$

property 2:-

$$x[n] \xrightarrow{Z.T.} X[Z] \qquad Roc\,\gamma_1 < |Z| < \gamma_2$$

$$x[-n] \xrightarrow{Z.T.} X[1/Z] \qquad \frac{1}{\gamma_2} < |Z| < 1/\gamma_1$$

Q. $u[n] = \dfrac{1}{1 - Z^{-1}}, \; |Z| > 1$

$u[-n] = \dfrac{-1}{1 - Z^{-1}}, \; |Z| < 1$

$u[-n-1] = \dfrac{1}{1 - Z}, \; |Z| < 1$

3) $x[n] \xrightarrow{Z.T.} X(Z)$

$$n x[n] \xrightarrow{Z.T.} -Z \frac{d}{dZ} X(Z)$$

Q. what is Z- transform of

$$x[n] = n a^n u[n].$$

Ans.

$$= -Z \frac{d}{dZ}\left(\frac{1}{1 - aZ^{-1}}\right) = -Z \frac{d}{dZ}\left(\frac{Z}{Z - a}\right)$$

$$= -z \frac{(z-a)\cdot 1 - z}{(z-a)^2}$$

$$= \frac{az}{(z-a)^2}$$

$$X(z) = \frac{az^{-1}}{(1-az^{-1})^2}$$

Q. What is

$$x[n] = na^{n-1} u[n].$$

$$x[n] = \frac{1}{a} \cdot n a^n u[n]$$

$$\downarrow$$

$$X(z) = \frac{z^{-1}}{(1-az^{-1})^2}$$

Q. $x[n] = n a^{n-1} u[n-1]$

$$= (n-1+1)\, a^{n-1}\, u[n-1]$$

$$x[n] = (n-1)\, a^{n-1} \cdot u(n-1) + a^{n-1} u(n-1)$$

$$= \frac{az^{-1}}{(1-az^{-1})^2} \times z^{-1} + \left(\frac{1}{1-az^{-1}} \times z^{-1}\right)$$

Q. $x[n] = a^{n-1} u(-n-1)$

$$= \frac{1}{a} \cdot a^n u(-n-1)$$

$$= \frac{1}{a} \cdot \left(-\frac{1}{1-az^{-1}}\right) \quad \text{Roc} : |z| < |a|.$$

Q. $x[n] = n a^{n-1} u(-n-1)$

$$= \frac{1}{a} \left[\underbrace{n a^n u(-n-1)}\right]$$

$$= \frac{1}{a} \times -z \frac{d}{dz} \times \left(\frac{z}{z-a}\right)$$

$$= \frac{1}{a} \times \frac{-az^{-1}}{(1-az^{-1})^2} = \frac{-z^{-1}}{(1-az^{-1})^2}, \quad \text{Roc} : |z| < |a|.$$

Condition for Causal System:-

1. If $h[n] = 0$ for $n < 0$
then System is called as causal system.

2. If $H(z)$ is form of rational function then:-
(i) Roc must be right of right most plane pole.

(ii) If $H(z) = \frac{N^r}{D^r}$

then for causal system degree of Numerator must be less than Equal to degree of Denominator.

Q. If $H(z) = \frac{z^3 - 2z^2 + z}{z^2 + z/4 + 1/8}$ → degree = 3 (numerator), degree = 2 (denominator)

Ans. So, Here degree of Numerator > degree of Dr.

So, it is Non-causal System.

~~Q. $H(z) = \frac{e^z}{z+1}$~~

Stability in Z-domain -

(i) for a stable system $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$

(ii) for $H(z)$ form to be stable, its Roc must include unit circle.

Q. $H(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} + \frac{1}{1 - 2z^{-1}}$

Roc: $|z| > 2$

Ans. As it does not include unit circle so must be unstable.

→ Here ROC is right of right most pole so It is causal system.

Q. $H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-2z^{-1}}$, $|z| < \frac{1}{2}$

Ans. It is not stable because ROC does not contain unit circle, Also Non-causal System, because ROC is not right of right most pole.

Q. $H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-2z^{-1}}$ ROC: $\frac{1}{2} < |z| < 2$

Ans Stable & Non Causal System.

Q. A causal LTI system is describe by difference
G-2004 Equation $2y[n] = \alpha y[n-2] - 2x[n] + \beta x[n-1]$

~~Ans.~~ A System is stable, only if

A. $|\alpha| = 2$, $|\beta| < 2$

B. $|\alpha| > 2$, $|\beta| > 2$

C) $|\alpha| < 2$, Any value of β (✓)

d) $|\beta| < 2$, any value of α.

Ans.

$$2Y(z) = \alpha z^{-2}Y(z) - 2X(z) + \beta z^{-1}X(z)$$

$$Y(z) = \frac{(\beta z^{-1}+2)}{(\alpha z^{-2}-2)} = \frac{(2-\beta z^{-1})}{(\alpha z^{-2}-2)}$$

$\alpha z^{-2} = 2$

$z^2 = \alpha/2$

$|z| = \sqrt{\alpha/2}$

$\sqrt{\alpha/2} < 1 \Rightarrow \alpha < 2$

[As ROC is depend on denominator not Nr. So Any value of β].