Structural Geology
and
Structural Analysis

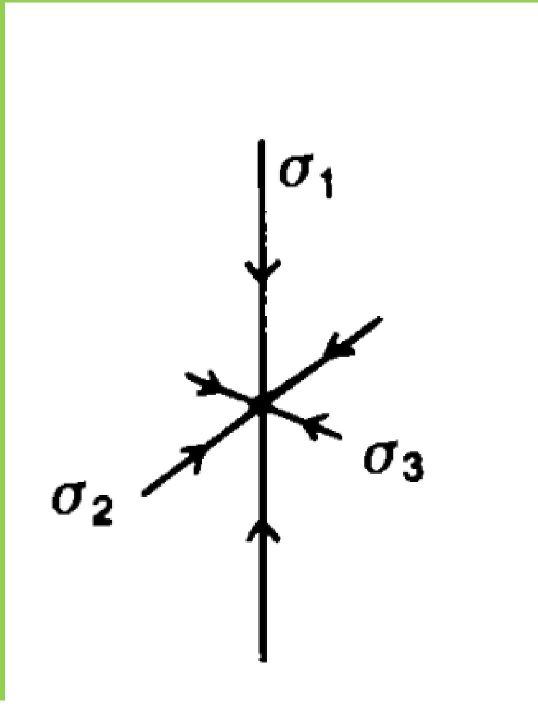
The Earth is a Dynamic Planet.

Principal Stresses and the Stress Axial Cross

The three mutually perpendicular planes on which the shear stress is zero are called principal stress planes, and the normal stresses across them are called the principal stress axes. These are given the conventional notation σ_1, σ_2 and σ_3, (where $\sigma_1, > \sigma_2 > \sigma_3$) or greatest, intermediate and least principal stresses, respectively.

Axial Cross

The three mutually perpendicular stress axes lengths may be drawn proportional to the magnitudes of the principal stresses



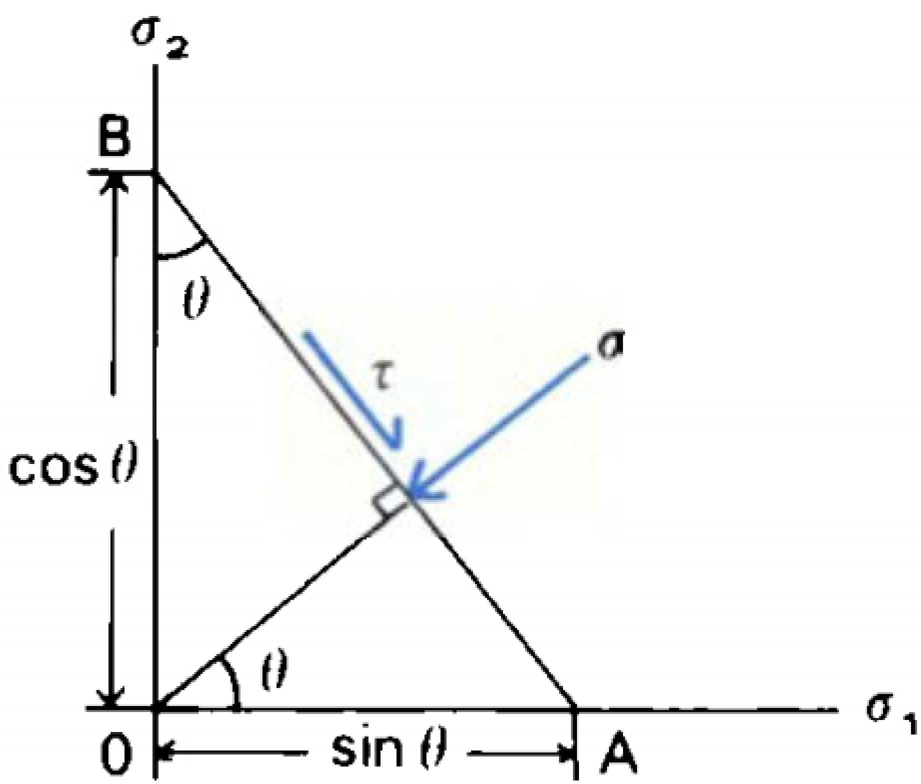
Stresses acting on a Given Plane:

If the principal stresses are known. the stresses acting on any plane with known orientation can be calculated.

Consider the stresses acting on a plane AB whose normal makes an angle θ (theta) with (σ_1 in a two-dimensional stress field with principal stresses σ_1 and σ_2 . Let the line AB represent unit length (one side of a square of unit area in three dimensions). Then OA = sin θ and OB = cos θ. The forces acting along OA and OB are thus (σ_1 cos θ and (σ_2 sin θ respectively (from force = stress x area). Resolving these forces perpendicular and parallel to the plane AB, the normal stress σ and shear stress τ are as follows:

$$\sigma = \sigma_1 \sin^2\theta + \sigma_2 cos^2\theta \qquad 1$$

$$\tau = \sigma_1 \sin^2\theta - \sigma_2 cos^2\theta \qquad 2$$



σ2
B
θ
τ
σ
cos θ
θ
σ1
0
sin θ
A

MAXIMUM SHEAR STRESS

The value of r in the last equation is a maximum when 2θ = 90° and sin 2θ = 1. Thus the planes of maximum shear stress make an angle of 45° with σ_1 and σ_2 regardless of the values of σ_1 and σ_2 .

In these positions,

$$\tau = 1/2(\sigma_1 - \sigma_2)$$

Hydrostatic and Deviatoric Stresses:

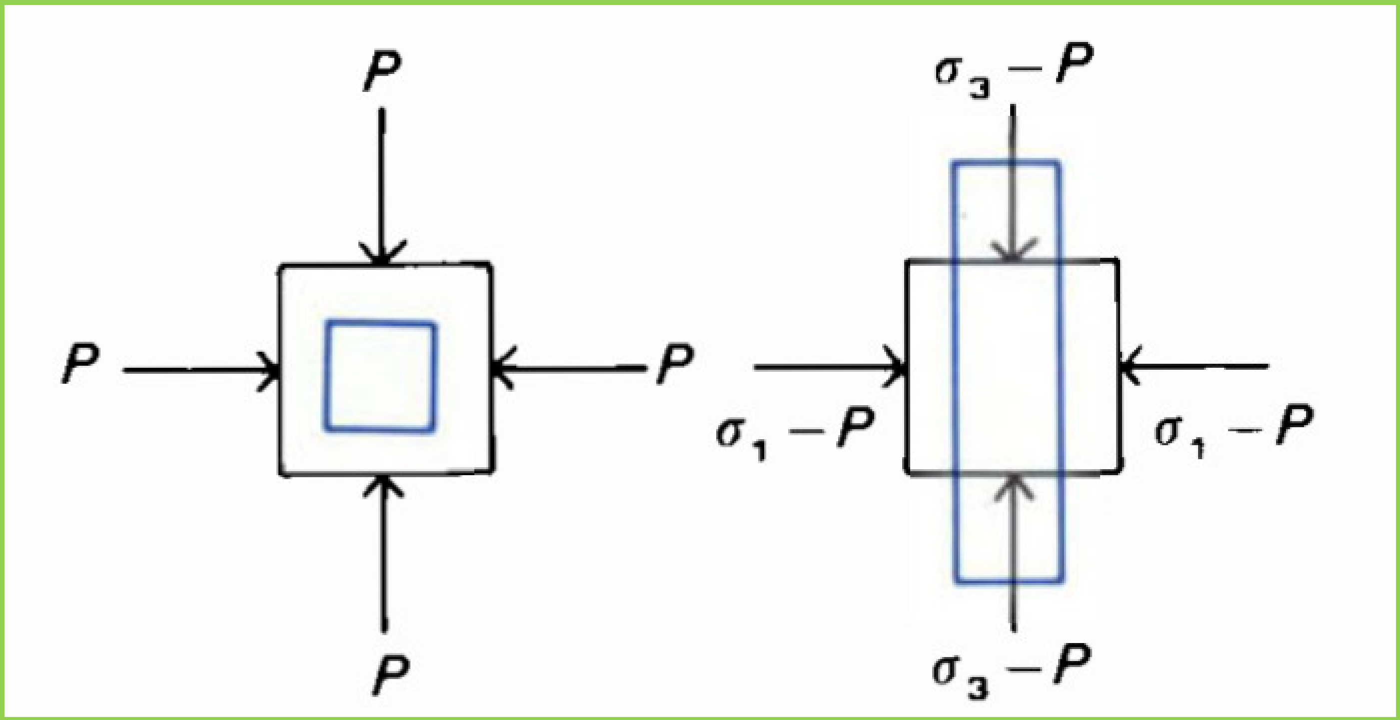
Where the principal stresses are equal, the state of stress is said to be hydrostatic, i.e. it corresponds to the stress state of a fluid. Hydrostatic stress will cause volume changes but not shape changes in a material

In a system with unequal principal stresses σ_1, σ_2 and σ_3, it is convenient to recognize a mean stress P, which represents the hydrostatic stress component of the stress field. Thus:

$$P = 1/3(\sigma_1 + \sigma_2 + \sigma_3,)$$

The deviatoric stress component, are σ_1-P , σ_2-P and σ_3 - P.

Deviatoric stresses cause a shape change

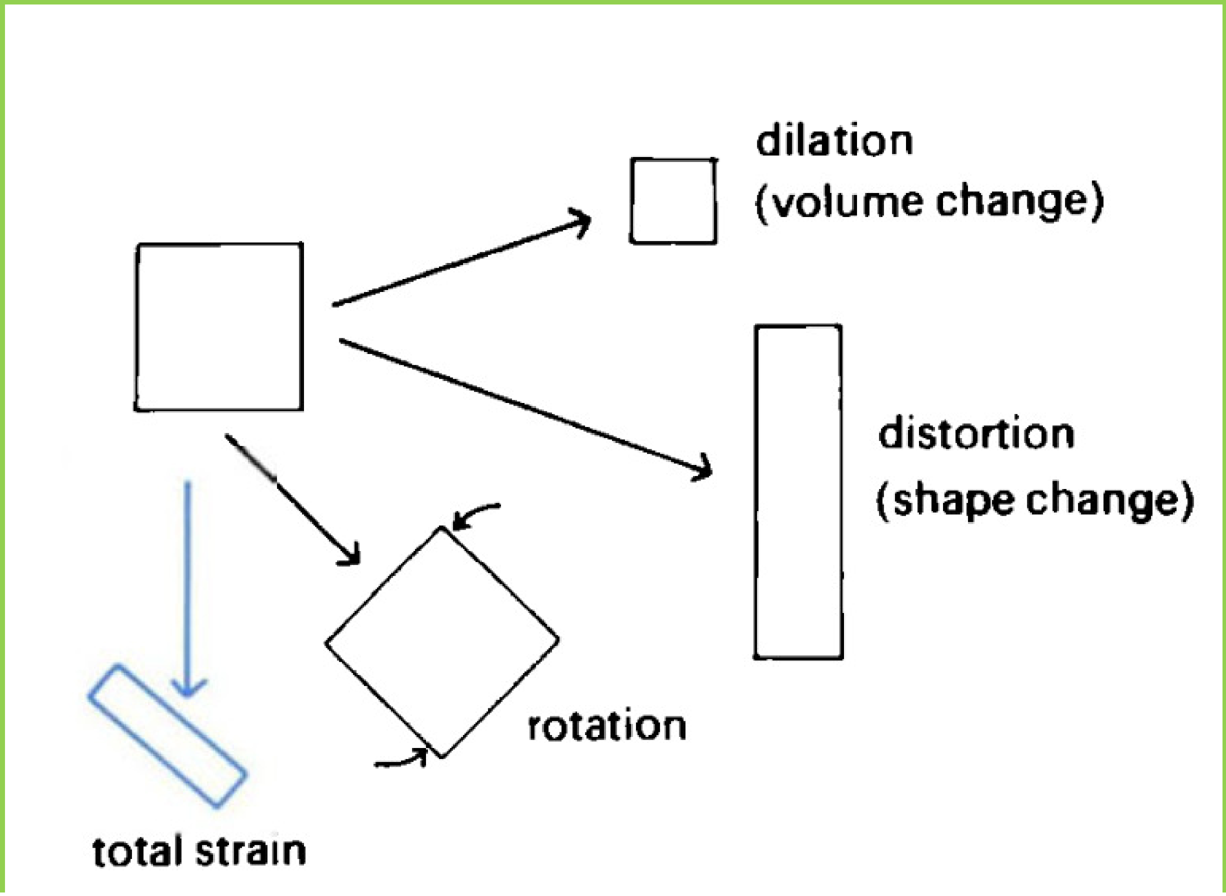


Strain

Strain results from non-rigid body deformation, either through a change in size (dilation) and/or a change in shape (distortion).

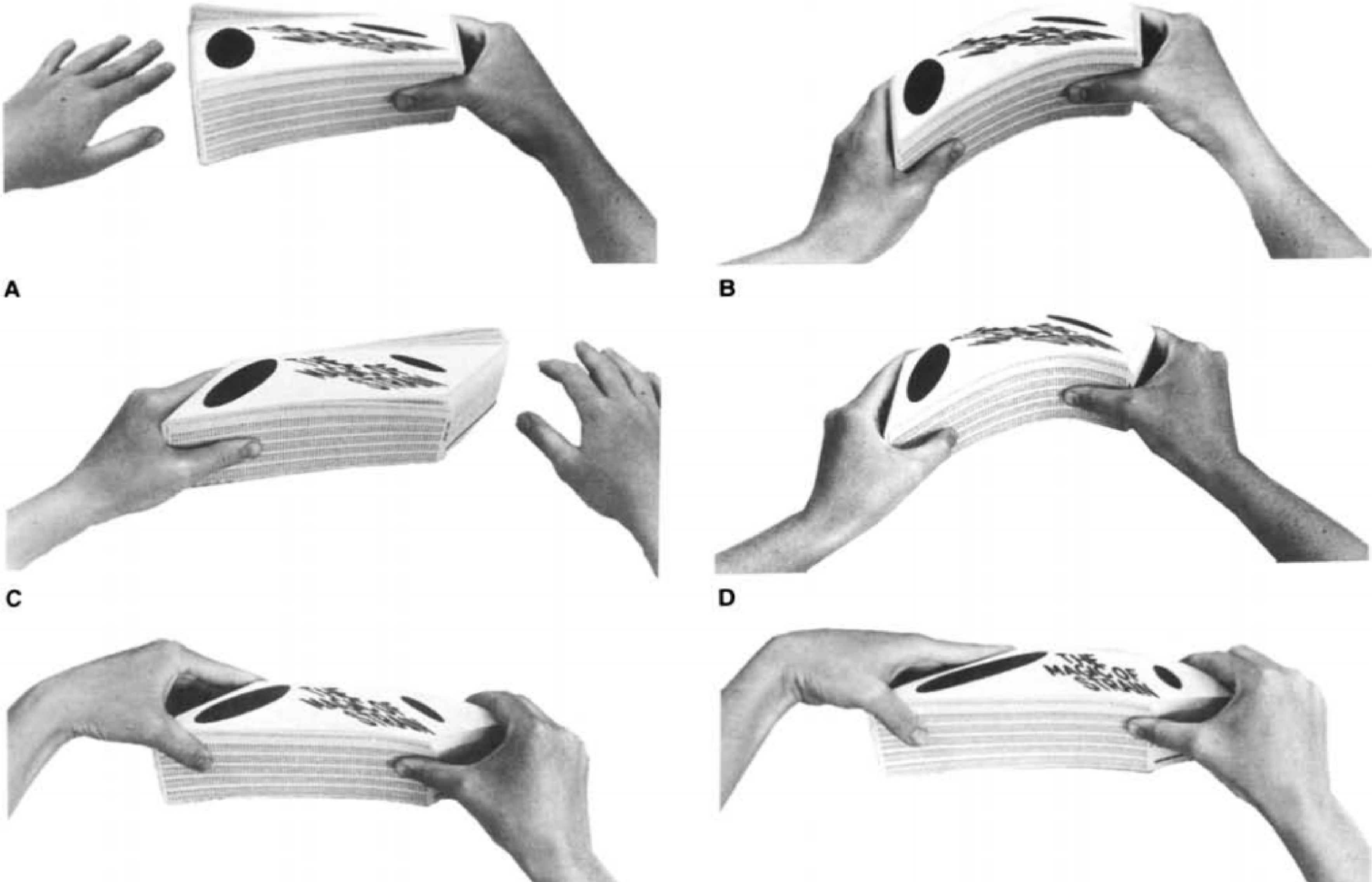
or

Strain is expressed as dilation (volume change) or distortion (shape change), or as a combination of these processes.



The nature of strain: dilation, distortion and rotation.

The Magic of Strain

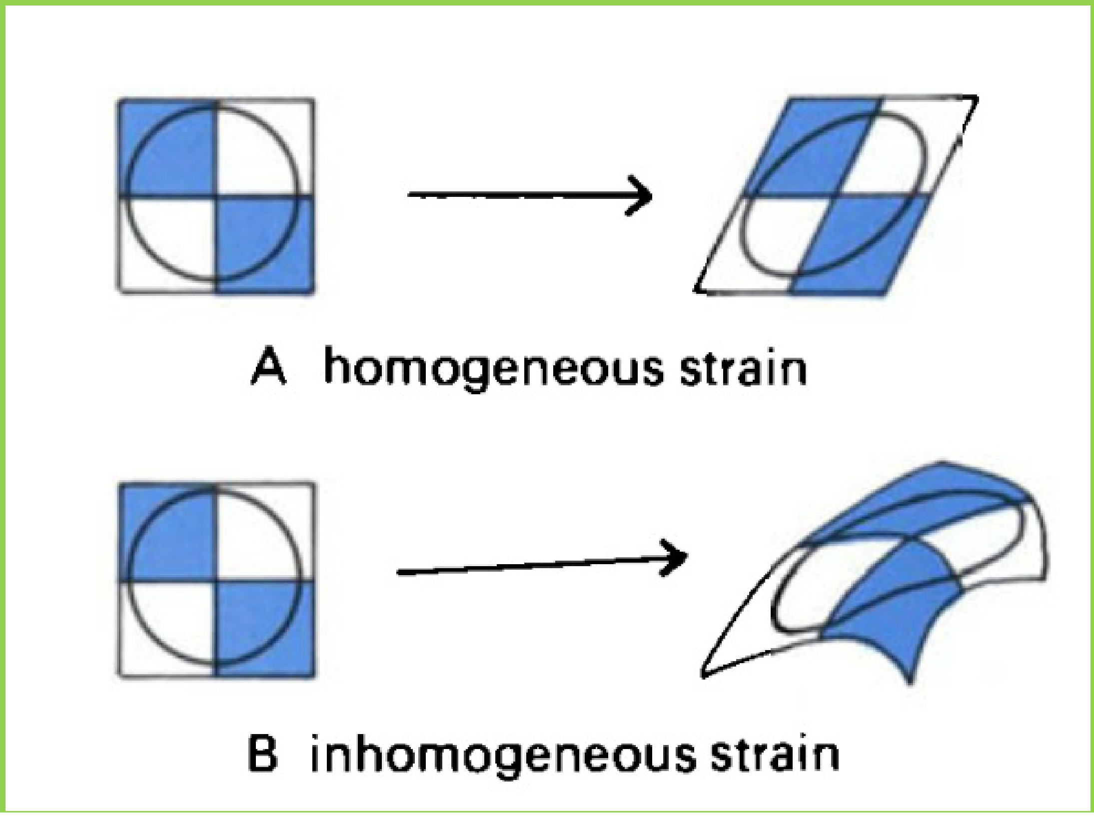


aspect ratio (long axis/short axis) or ellipticity (ratio between the long and short axes of the ellipse)

Homogeneous and heterogeneous deformation

For homogeneous deformation, originally straight and parallel lines will be straight and parallel also after the deformation. The strain and volume/area change will be constant.

The criteria for inhomogeneous (heterogeneous) strain are thus that straight lines become curved and that parallel lines become non-parallel.



Measurement of Strain:
Strain may be measured in two ways: either by a change in length of a line (linear strain, or *extension*) or by a change in the angle between two lines (*angular strain, or shear strain*)

extension, symbolized by ***e;*** the other is **stretch,** symbolized by *S*
Consider line L whose original length (l_0) is 5 cm During deformation, the non rigid-body in which L is contained changes shape and/or size such that the line stretches to a final length (l_f) of 8 cm .The change in length (Δl) is 3 cm. The magnitude of extension (e) in the direction of lengthening is the change in unit length of the line.

$$e = \frac{l_f - l_0}{l_0}$$

$$e = (8 \text{ cm} - 5 \text{ cm})/5 \text{ cm} = 0.6$$

A 0.6 value for extension ***e*** corresponds to a 60% lengthening of the line. Percent lengthening (or *percent shortening*) is determined by multiplying ***e*** by 100%.

Stretch is equal to final length (l_f) divided by original length (l_0), which is also equal to the value of extension plus one (i.e., $1 + e$).

$$e = \frac{l_f - l_0}{l_0}$$

$$e = \frac{l_f}{l_0} - 1$$

$$e + 1 = \frac{l_f}{l_0}$$

$$S = \frac{l_f}{l_0} = 1 + e$$

Quadratic elongation (Lambda (λ)) is the square of the stretch.

$$\lambda = S^2$$