B. Tech Sem-I, Tutorial -II, 17 Sep 2025

Differentiability

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Remember the following results (1-5):

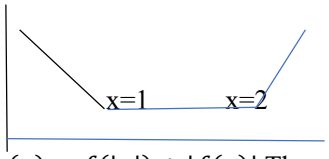
1. A necessary condition for the existence of a finite derivative. Continuity is necessary but not a sufficient condition for the existence of a finite derivative. **(CO1,L1)**
2. Intermediate Value Theorem for differentiability (**Darboux Theorem**). If f is finitely differentiable in $[\,a,b]$ and $f'(a), f'(b)$ are of opposite sign, then there exists at least one point $c \in]a,b[$ such that $f'(c) = 0$. **(CO1,L1)**
3. **Rolle's Theorem** : If a function $f(x)$ is such that
   (i) $f(x)$ is continuous in [a,b],
   (ii) $f(x)$ exists for every point in]a,b[,
   (iii) $f(a) = f(b)$, then ∃ at least one value of x (say c) ∈]a, b[such that $f'(c) = 0$ **(CO2,L3)**
4. **Lagrange's Mean Value Theorem (LMVT) (First Mean Value Theorem)** :If a function $f(x)$ is
   (i) Continuous in [a,b] and (ii) Differentiable in (a,b) then ∃ at least one $c \in$ (a, b) such that $\frac{f(b)-f(a)}{b-a} = f'(c)$. **(CO2,L3)**
5. **Cauchy's mean value theorem (CMVT) (Second Mean Value Theorem)**: If two functions $f(x)$ and $g(x)$ are
   (i) $f(x)$, g(x) are continuous in [a,b],
   (ii) $f(x), g(x)$ both are differentiable in (a,b)
   (iii) $g'(x) \neq 0$ for all x in (a,b)

   then ∃ at least one value of x (say c) ∈]a, b[such that $\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$, **(CO2, L3)**
6. Define differentiability and explain Left Hand and Right Hand derivatives**. (CO1, L1)**
7. Analyze the differentiability of $f(x) = |\,x\,| + |x-1|$ **(CO1,L4)**
8. A function $f(x) = x^p \cos(\frac{1}{x})$, $if\ x \neq 0$; and $f(0) = 0$. What conditions should be imposed on p so that $f(x)$ can be made (i) continuous at x = 0 (ii) Differentiable at x = 0. **(CO_1,L4)**
9. Can you interpret Rolle's Theorem Geometrically and Algebraically? (**CO1, L1,L2)**
10. Discuss the applicability of Rolle's Theorem for $f(x) = 2 + (x-1)^{\frac{2}{3}}$ in [0,2]. (**CO2,L3)**
11. Verify Rolle's Theorem for $f(x) = 2x^3 + x^2 - 4x - 2$. (**CO2,L3)**
12. Show that ∃ at least one root of $e^x sinx - 1 = 0$ between any two roots of $e^x cosx = 1$. (**CO2,L3)**
13. Give Geometrical interpretation of Lagrange's Mean Value Theorem. (**CO1,L1,L2)**

Some important deductions you can derive from mean value theorems.

14. If f (x) is continuous on [a,b] , differentiable on (a,b)and if $f'(x) = 0$ for all x in] a,b [,then $f(x)$ is constant throughout [a,b]. (**CO2,L3)**
15. If $f(x)$ and $\emptyset(x)$ are continuous functions on [a,b], differentiable on]a,b[also $f'(x) = \emptyset(x)$ in]a,b[,then $f(x)$ and $\emptyset(x)$ differ only by a constant. (**CO2,L3)**
16. If $f'(x) = k$ for each point x of [a,b] , k being a constant , then $f(x) = kx + C$ for all x in [a,b] , where C is a constant . (**CO2,L3)**
17. If f is continuous on [a,b] , and $f'(x) \geq 0$ in] a,b [, then f is increasing in [a,b]. (**CO2,L3)**
18. If $f(x) = (x-1)(x-2)(x-3)$ and a = 0 , b = 4, find c using LMVT. (**CO2,L3)**
19. Show that any chord of parabola $y = Ax^2 + Bx + C$ is parallel to the tangent at the point whose abscissa is same as that of the middle point of the chord. (**CO2,L3)**
20. Compute the value of θ in the first mean value theorem
    $f(x+h) = f(x) + hf'(x+\theta h)\ if\ f(x) = ax^2 + bx + c$ **(CO2,L4)**
21. Use Lagrange's mean value theorem to prove that $1 + x < e^x < 1 + xe^x \quad \forall x > 0$. (**CO2,L3)**
22. Verify Cauchy's mean value theorem for x^2 and x^3 in the interval [1,2]. (**CO2,L3)**
23. For CMVT, insert $f(x) = e^x$ and $g(x) = e^{-x}$, and show that 'c' is the arithmetic mean of a and b. (**CO2,L3)**

24. Let R be the set of real numbers and f: R $\rightarrow R$ such that for all x and $y\ in\ R$, $|f(x)-f(y)| \leq |x-y|^3$. Prove that $f(x)$ is a constant function. **(CO3, L4)**
25. If $f(x) = x^2 \sin\left(\frac{1}{x^2}\right)$, if $x \neq 0$ and 0 if $x = 0$, then $f(x)$ is (a) discontinuous at $x \neq 0$ (b) continuous at $x = 0$ (c) Discontinuous at $x = 0$, (d) None of these **(CO1,L2) Ans: (b)**
26. Which of the among (a) $f(x)$ is finitely differentiable in [a,b] and $f'(a)$ and $f'(b)$ are of opposite signs, then $\exists$ at least one $c \in (a,b)\ such\ that\ f'(c) = 0$ (b) Every differentiable function is continuous (c) Every continuous function is differentiable (d) Every continuous function is bounded; is not true **Ans: (c) (CO1,L2)**
27. If $f(x) = |x|$ Then (a) $f(x)$ is continuous but not differentiable at $x = 0$ (b) is continuous as well as differentiable at $x = 0$ (c) is differentiable at $x = 0$ but not continuous at $x = 0$ (d) None of these **Ans: (a) (CO1,L2)**
28. If adjoining graph of a function $f(x)$ Then (a) $f(x)$ is continuous at $x = 1,2$ but not differentiable (b) $f(x)$ is differentiable at $x = 1,2$ but not continuous (c) is differentiable and continuous at $x = 1,2$ (d) None of these **Ans: (a) (CO1,L2)**



29. If $f(x) = \begin{cases} -1 & -2 \leq x \leq 0 \\ x-1 & 0 < x \leq 2 \end{cases}$ and $g(x) = f(|x|) + |f(x)|$ Then $g(x)$ is (a) Differential everywhere in (-2,2) (b) not differential in (-2,2) but continuous at $x = 0,1$ (c) differential at $x = 1,2$ (d) differential everywhere in (-2,2) except at $x = 0,1$ **Ans: (d) (CO1,L1,L2)**
30. Let $f(x)$ be an even function and $f'(0)$ exists then the value of $f'(0)$ is (a) 1 (b) ∞ (c) 0 (d) None of these **Ans(c) (CO1,L1,L2)**
31. If $f(x) = \begin{cases} x\tan^{-1}\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$ Then $f(x)$ is (a) Continuous at 0 but not differentiable (b) is continuous as well differentiable at 0 (c) Neither differentiable nor continuous at 0 (d) None of these **Ans: (a) (CO1,L2)**
32. $f(x) = \frac{\tan(\pi\ |x-\pi\ |}{1+\ |x\ |^2}$ and $g(x) = \frac{x}{1+\ |x\ |}$ (a) are continuous for all real x (b) are differentiable for all real x (c) not continuous but differentiable for all real x (d) None of these **Ans: (b) - (CO1,L2)**
33. Prove that $f(x) = x^2 \sin\left(\frac{1}{x}\right)$ for $x \neq 0\ and\ f(0) = 0$ is differentiable at $x = 0$. **(CO3,L4)**
34. Discuss the differentiability of the function $y = x - 1\ |+\ |x-2\ |$ in the interval [0,3].- **(CO3,L4)**
35. If function $f(x)$ is defined as $f(x) = 2x - 3, if\ 0 \leq x \leq 2$
$= x^2 - 3,\ 2 < x \leq 4$, show that the function $f(x)$ is continuous but not differentiable at $x = 2$. **(CO3,L4)**
38. Verify LMVT for $f(x) = 2x^2 - 7x + 10$, $x \in [2,5]$. **(CO3,L3)**
39. Verify CMVT for $f(x) = x^2 - 2x + 3,\ g(x) = x^3 - 7x^2 + 26x - 5$ in [-1,1]. **(CO3,L3)**
40. Show that $f(x) = \frac{x\left(e^{1/x} - e^{-1/x}\right)}{\left(e^{1/x} + e^{-1/x}\right)}$, $x \neq 0$ and $0, if\ x = 0$ is not differentiable at $x = 0$. **(CO3,L5)**