Tutorial-1

1. Find the envelope of the circles drawn upon the radii vectors of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ as diameter.
2. Show that the envelope of the circles whose centres lie on the parabola $y^2=4ax$ and which pass through its vertex is the cissoids $y^2(2a+x)+x^3=0$
3. Circles are described on the double ordinates of the parabola $y^2=4ax$ as diameters: prove that their envelope is the parabola $y^2=4a(x+a)$
4. Show that the envelope of the circles who we centres lie on the rectangular hyperbola $x^2-y^2=a^2$ and which pas through the origin is the lemniscuses $r^2=4a^2\cos 2\theta$.
5. Show that the envelope of the circles described n the central radii of a rectangular hyperbola as diameters is a lemniscuses

$$r^2=a^2\cos 2\theta$$

6. From any point of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. perpendiculars are drawn to the axes, and the feet of the perpendiculars are joined. Show that the straight line thus formed touches the curve

$$\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1.$$

7. Show that the envelope of the poplars of points on the ellipse $\frac{x^2}{h^2}+\frac{y^2}{k^2}=1$ with respect ot the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{h^2x^2}{a^4}+\frac{k^2y^2}{b^4}=1$

8. Find the envelope of a family of parabolas of given latus rectum and parallel axes when the locus of their foci is a given straight line $y = px + q$.
9. Show that the envelope of the straight line joining the extremities of a pair of conjugate diameters of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ is ellipse } \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{2}$$

10. Find the envelopes of circles described on the radii vectors of the following curves as diameters
    (i) $\frac{l}{r} = 1 + e\cos\theta$,
    (ii) $r^3 = a^3\cos 3\theta$,
    (iii) $r\,cos^n\left(\frac{\theta}{n}\right) = a$.
11. Find the envelopes of the straight lines drawn at right angles to the radii vectors of the following curves through their extremities.
    (i) $r = ae^{0\cot\alpha}$,
    (ii) $r^n = a^n\cos n\theta$,
    (iii) $r = a + b\cos\theta$.
12. Find the envelope of the straight line $\frac{x}{a} + \frac{y}{b} = 1$, where the parameters a and b are connected by the rlation $a^n + b^n = n^n$, c being a costant.
13. Find the envelope of the straight line $\frac{x}{a} + \frac{y}{b} = 1$ when $a^m + b^n = c^{m+n} = c$ being a constant

14. Prove that the envelope of the ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$having the sum of their semi- axes constant and equal to c, is the as toroid $x^{2/3}+y^{2/3}=a^{2/3}$.
15. Find the envelope of the family of straight lines $\frac{x}{a}+\frac{y}{b}=1$,

    Where a, b are cibbected by the relation

    (i) $a+b=c$

    (ii) $a^2+b^2=c^2$

    (iii) $ab=c^2, c$ is a constant
16. Find the envelope of the system of concentric and coaxial ellipses of constant are

    [Hint. Taking the common centre as robin and the common axes as coordinate axes, let the equation of the family of ellipses be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. The area of an ellipse is πab. Since the ellipses are of constant area, so let $ab=c^2$, where c is a constant. Now find the envelope.]
17. Show that the envelope of the family of parabolas

$$\left(\frac{x}{a}\right)^{1/2}+\left(\frac{y}{b}\right)^{1/2}=1,$$

    under the condition $ab=c^2$, is a hyperbola whose asymptotes coincide with the axes.
18. A straight line of given length slides with its extremities on two fixed straight lines at right angles. Find the envelope of the circle drawn on the sliding line as diameter

Answer Key

1. $(x^2+y^2)^2=a^2x^2+b^2y^2$

8. $p^2x - py + (pq + ap^2 + a) = 0$, Where 4a is the given latus tectum.

10. (i) $r^2(e^2 - 1) - 2l\, e\, r\cos\theta + l^2 = 0$

(ii) $r^{3/4} = a^{3/4}\cos(3\theta/4)$.

(iii) $r\, cos^{\,n-1}\{\theta/(n-1)\} = a$.

11. (i) $r\sin\alpha = ae^{\left(\alpha-\frac{\pi}{2}\right)\cot\alpha}e^{\theta\cot\alpha}$

(ii) $r^{n/(1-n)} = a^{m/(1-n)}\cos\{n\theta/(1-n)\}$

(iv) $r^2 - 2br\cos\theta + (b^2 - a^2) = 0$

12. $x^{n/(n+1)} + y^{n/(n-1)} = c^{n/(n+1)}$

13. $\{(m+n)^{m+n}x^m y^n\}/m^m n^n = c^{m+n}$

15. (i) $x^{1/2} + y^{1/2} = c^{1/2}$

(ii) $x^{2/3} + y^{2/3} = c^{2/3}$

(iii) $xy = c^2/4$

1. $4x^2y^2 = c^4$

18. A circle

Tutorial -2

1. Find the envelope of the straight lines $\left(\frac{x}{a}\right)\cos\theta + \left(\frac{y}{b}\right)\sin\theta = 1$, the parameter being θ and interpret the result geometerically.

   Find the envelope of the following families of straight lines

2. $y = m^2x + \left(\frac{1}{m^2}\right)$, the parameter being g m.
3. $y = mx + a\sqrt{(1+m^2)}$, the parameter being m.
4. $y = mx + am^3$, the parameter being m.
5. $y = mx + am^p$, the parameter being m.
6. $x\,cosec\,\theta - y\cot\theta = c,$ the parameter being θ
7. $x\,xos^{\,3}\,\alpha + ysin^3\alpha = a$, the parameter being α.
8. Find the envelope of the family of circles

$$x^2 + y^2 - 2ax\cos\alpha - 2ay\sin\alpha + c^2 = 0, (a^2 > c^2)$$

   Where α is the parameter, and interpret the result.

9. Find the envelope of the family of straight lines $\frac{s}{a} + \frac{y}{b} = 1$, where the two parameteras a and b are conneted by the relation $a^n + b^n = c^n, c$ being a constant.
10. Find the envelope of the following families of circles
    (i) $(x-\alpha)^2 + y^2 = 4\alpha,$ α being the parameter
    (ii) $(x-\alpha)^2 + (y-\alpha)^2 = 2\alpha, \alpha$ being the parameter

(iii) $(x-c)^2+y^2=R^2$, where c is the parameter

11. Find the envelope of the family of parabolas $y^2=m^2(x-m), m$ being the parameter
12. Find the envelope of the family of parablas$tx^2=t^2y=a$, the parameter being t
13. Find the envelope of the family of curves given by $\frac{x^2}{\alpha^2}+\frac{y^2}{k^2-\alpha^2}=1$, where α is the parameter
14. Find the envelope fo the family of curves

$\left(\frac{a^2}{x}\right)\cos\theta-\left(\frac{b^2}{y}\right)\sin\theta=\frac{c^2}{a}, \theta$ being the parameter.

15. Find the envelope of the family of straight lines

$x\,cos^{\,n}\theta+y\,sin^n\theta=a$, for different values of θ.

16. Find the envelope of the ellipse $x=a\sin(\theta-\alpha), y=b\cos\theta$, where α is the parameter.
17. Projectiles are fired from a gun with a constant intitial velocity v_0. Supposing the gun can be given any elevation and is kept always in the same vertical plane, what is the envelope of all possible trajectories, assuming their equation to be

$$y=x\tan\alpha-\frac{1}{2}\frac{g^x2}{v_0^2\,cos^2\alpha}?$$

18. Find the envelope of the straight lines $x\cos\alpha+y\sin\alpha=l\sin\alpha\cos\alpha$, where α is the parametr. Give the geometrixcal interpretation
19. Find the envelope of he family of straight liens

$$x\cos m\alpha+y\sin m\alpha=a\,(\cos n\alpha)^{m/n}$$

where α is the parameter

[Hint. Change the equation to polar co-ordinates by substituting

$x = r\,xos\,\theta, y = r\sin\theta$]

20. Show that the radius of curvature of the envelope of the lien

$x\cos\alpha + y\sin\alpha$ is $f(\alpha) + f''(\alpha)$

21. If $x^{2/3} + y^{2/3} = k^{2/3}$ is the envelope of the liens $\frac{x}{a} + \frac{y}{b} = 1$, the find the necessary relation between a, b and k.

22. Find the envelope of the family of cures $x^2\sin\alpha + y^2\cos\alpha = a^2$ where α is the parameter

23. Find the envelope of he family of curves $(y-c)^{\,2} - \frac{2}{3}(x-c)^3 = 0$, where c is the parameter.

Answer Key

1. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Each line of the given family is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

2. $y^2 = 4x$

3. $x^2 + y^2 = a^2$

4. $4x^2 + 27ay^2 = 0$

5. $(p-1)^{p-1}x^p + p^p a y^{p-1} = 0$

6. $x^2 - y^2 = e^2$

7. $a^2(x^2 + y^2) = x^2y^2$

8. $(x^2 + y^2 + c^2)^2 = 4a^2(x^2 + y^2)$. Circles with centre at orgin and radii $a \pm \sqrt{(a^2 - c^2)}$

9. $x^{1/2} + y^{1/2} = c^{1/2}$

10. (i) $y^2 - 4x - 4 = 0$; (ii) $(x + y + 1)^2 = 2(x^2 + y^2)$; (iii) $y = \pm R$

11. $4x^3 = 27^{y^2}$
12. $x^6 + 4ay = 0$
13. $x \pm y = \pm k$
14. $(a^2/x)^2 + (b^2/y)^2 = (c^2/a)^2$
15. $x^{2/(2-n)} + y^{2/(2-n)} = a^{2/(2-n)}$
16. $x = \pm a.$
17. $y + \frac{\left(\frac{1}{2}gx^2\right)}{v_0^2} = \frac{v_0^2}{(2g)}$
18. $x^{2/3} + y^{2/3} = l^{2/3}$
19. $r^{n/(m-n)} = a^{n/(m-n)} \cos\{n\theta/(m-n)$, Where r, θ are the polar coordinates of (x, y)
21. $a^2 + b^2 = k^2$
22. $x^4 + y^4 = a^2$
23. $x - y = 0, x - y = \frac{2}{9}$