

DEPARTMENT OF MATHEMATICS
V B S PURVANCHAL UNIVERSITY
JAUNPUR-222001 (U.P.) INDIA



Syllabus Of
MATHEMATICS
For Two Years P.G. Programme
Of the Constituent Colleges w. e. f. 2022-2023 onwards



Submitted By:
Convener(s)/Member(s), B O S
of

P.G. Mathematics

Outline for the Semester Course of P.G. (Mathematics)							
Semester-VII							
S.No.	Paper	Course Code	Paper Title	Nature of Paper	Credits	Marks	
1.	I	B030701T	Algebra-I	Theory	4	100=25+75	
2.	II	B030702T	Real Analysis	Theory	4	100=25+75	
3.	III	B030703T	Point-Set Topology	Theory	4	100=25+75	
4.	IV	B030704T	Partial Differential Equation and Calculus of Variations	Theory	4	100=25+75	
5.	V	B030705P	Programming in C	Practical	4	100=25+75	
6.	VI		Minor Elective (Other than own faculty)		4/5/6	Non-Gradial	
7.	VII	B030706R	Topic and Supervisor Allotment for the Project Work/ Survey/Industrial Training				
Semester-VIII							
S.No.	Paper	Course Code	Paper Title	Nature of Paper	Credits	Marks	
1.	I	B030801T	Complex Analysis	Theory	5	100=25+75	
2.	II	B030802T	Functional Analysis	Theory	5	100=25+75	
3.	III	B030803T	Classical Mechanics	Theory	5	100=25+75	
4.	IV	B030804T	Elective Any one of the following	Theory	5	100=25+75	
Fourier Analysis and Summability Theory Differential Geometry Extended MATLAB Special Theory of Relativity							
5.	V	B030805R	Evaluation of the Project Work/ Survey/Industrial Training by Progress report (in hand written/Printed 20 or more pages), Presentation & Viva-Voice		8	100	
Semester-IX							
S.No.	Paper	Course Code	Paper Title	Nature of Paper	Credits	Marks	
1.	I	B030901T	Algebra-II	Theory	4	100=25+75	
2.	II	B030902T	Integral Equation and Boundary Value Problems	Theory	4	100=25+75	
3.	III	B030903P	Wavelet	Practical	4	100=25+75	
4.	IV	B030904T	Elective-I Any one of the following	Theory	4	100=25+75	
Advanced Discrete Mathematics Advanced Linear Algebra Fluid Mechanics Probability Theory							
5.	V	B030905T	Elective-II Any one of the following	Theory	4	100=25+75	
Advanced Ordinary Differential Equation Fuzzy Sets and Its Applications Non linear Analysis Special Function							
6.	VI	B030906R	Topic and Supervisor Allotment for the Project Work/ Survey/Industrial Training May be same as Previous Allotment				
Semester-X							
S.No.	Paper	Course Code	Paper Title	Nature of Paper	Credits	Marks	
1.	I	B031001T	Measure and Integration	Theory	5	100=25+75	
2.	II	B031002T	Elective-I Any one of the following	Theory	5	100=25+75	
Advanced Topology Algebraic Number Theory Mathematical Modelling Advanced Partial Differential Equations							
3.	III	B031003T	Elective-II Any one of the following	Theory	5	100=25+75	
Advanced Special Function Mathematical Method Module Theory Wavelet Analysis							
4.	IV	B031004T	Elective-III Any one of the following	Theory	5	100=25+75	
Differential Geometry of Manifolds Dynamical System General relativity and Cosmology Operator Theory							
5.	V	B031005R	Evaluation of the Project Work/Survey / Industrial Training by Presentation & Submission of Thesis in hand written/Printed 30 or more pages and Viva-Voice.		8	100	

Proposed by BoS Committee

Dated: June 14, 2022

**The Full Structure of the Semester Courses for the
P.G. Mathematics With Detailed Syllabus
for the Constituent Colleges of
Veer Bahadur Singh Purvanchal University
Jaunpur, Uttar Pradesh.**

1. The course of **P.G.**(Mathematics) will be spread in two years and four semester namely- Semester-VII, Semester-VIII, Semester-IX and Semester-X. Each years consistent of two semester namely even semester and odd semester. Odd semester (Semester-VII and Semester-IX) will moves during the period 15th July to 15th December and Even semester (Semester-VIII and Semester-X) will moves during the period 1st January to 31st May including examinations also.

2. Pogramme Specific Outcomes (PSOs)

PS01. To develop deep understanding of the fundamental axioms in mathematics and capability of developing ideas based on them.

PS02.To provide advanced knowledge of topics in pure mathematics particularly in Analysis and Geometry empowering the students to proceed with the area at higher level.

PS03.To develop understanding of applied mathematics and motivating the students to use mathematical techniques as a tool in the study of other scientific domains.

PS04.To encourage students for research studies in Mathematics and related fields.

PS05.To provide students a wide variety of employment options as they can adopt research as a career or take up teaching jobs or can get employment in banking or can go for any other profession.

PS06.To inculcate problem solving skills, thinking and creativity through presentations, assignments and project work.

PS07.To help students in their preparation (personal counselling, books) for competitive exams e.g. NET, GATE, etc.

PS08.To enable the students being life-long learners who are able to independently expand their mathematical expertise when needed.

3. Minimum eligibility requirement for **The Course** of two year or four semester **P.G.**(Mathematics): Students must have passed a three-year degree course with Mathematics as a major subject.
4. There will be five /Six / papers in each semester with Practical/ viva-voice or Dissertation/project work examinations, Practical's in semester-VII and Semester-IX(odd semester) and Dissertation/project work's in Semester-VIII and Semester-X (even semester).
5. The University adopted the semester and credits based courses. One Credit is defined as the one/two/three lecture of theory/practical/field work per week respectively.
6. A total of 30-75 (Duration of one Lecture=60 Minutes) lectures plus five periods of the interactive from are to be devoted to each paper. Four to Six lectures per week are to be allotted to each paper.
7. Each Theory papers (Other than Practical Subject) will be of 4/5 credits (Max Marks=75 marks) except where stated otherwise. Duration for examination of a paper will be 3 hours.
8. **Examination and Evaluation System**
 Courses with only Theory (25%Internal + 75%External)
 Courses with Theory and Practical (50%Practical + 50%Theory External)
 Courses with only Practical (25%Internal + 75%Practical)
9. Registered students are required to take a minor elective(Other than own faculty) **Non-Gradial** course of 4/5/6/credits during the first or second semester to complete the P.G. Mathematics Programme.
Non-Gradial Course's: It means a course which is compulsorily registered by the student for the completion of P.G. Mathematics Programme. The non gradial course will be evaluated as satisfactory or not-satisfactory (according as the marks obtained in that course's \geq or $<$ 50% Marks). The marks obtained by the student in a non-gradial course will not be taken into account for calculating Grade/OGPA/CGPA/Percentage/etc. If a student secures **Not-Satisfactory**, he shall be repeat the course whenever the university offers it.
10. There will be 25 percentage (Max Marks=25) internal evaluation in each courses with only Theory/only Practical papers (Other than Theory and Practical Subject) based on

Evaluation of Internal Marks (25 %) based on the

Regular Student			Private Student
1. Attendance	5 Marks	Remark (if any)	-
i. 91-100 (in percentage)	5 Marks	-	-
ii. 81-90 (in percentage)	4 Marks	-	-
iii. 75-80 (in percentage)	3 Marks	-	-
2. Class test	15 Marks	-	15 Marks
3. Assignment	5 Marks	-	10 Marks
Total Marks	25 Marks	-	25 Marks

11. **Format of the Question Paper:** The Question paper divided in to three section namely Section *A*, Section *B* and section *C*. There will be one compulsory question in this Section *A* consisting of 10 parts of short answer type questions based on the whole course, out of which all parts will have to be answered. Each question in section *A* have 2 marks. The section *B* contained Eight question, there will be 16 parts from whole course (at least two question/four parts from each unit), each question contains two parts of equal marks, out of which five questions will have to be answered. Each question in section *B* have 7 marks. The section *C* contained 4 question (at least one for each unit) out of which 2 questions will have to answered. Each question have 10 marks. Thus in all, 8 questions will have to be attempted out of 13 questions will have to be set, except stated otherwise.
12. The Departmental Committee/ Department shall assign a topic for Project Work/ dissertation/Survey along with an supervisor to a Candidate in the beginning of the semester-VII and Semester-IX.
13. The Progress and submission of Project Work/ dissertation/Survey Evaluated by Viva-Voice in the end of Semester-VIII and Semester-X.
14. The Project Work Examination of 100 marks will be held during Semester-VIII and Semester-X. The Board of Examiners will consist of one External and one/two internal examiners(supervisor) recommended for appointment by the BoS/HoD of department. The title of project work will be given by internal examiners which are fixed by beginning of courses in respective semester. Under the project, the candidate will present a dissertation in his/ her own handwriting or printed. The dissertation will consist of at least one theorem/article with proof and one problem with solution, relevant definitions with examples and/or counter examples, where necessary, from the paper of Mathematics studied in previous or current semesters according respective supervisor.
15. An evaluation of project work there will be a board of examiners consisting of an external examiner and an internal examiner(supervisor). The dissertation will be forwarded by the principal/HoD of the college/university centre.
16. The Practical and Theory Subject will be of 4(2+2) credits (Max Marks 100=50+50) except where stated otherwise.

Evaluation of Dissertation/ Project Work /Industrial Training/ Survey/ Viva-voice Examination will be of 100 marks based on

Regular Student			Private Student
1. Attendance	10 Marks	Remark (if any)	NA
i. 91-100 (in percentage)	10 Marks		
ii. 81-90 (in percentage)	08 Marks	-	-
iii. 75-80 (in percentage)	05 Marks		
2.Conferences/Seminar/Workshop Attended in the Related topic / Inter Disciplinary State/University Label	02 Marks	Additional Marks	02 Marks
3.Conferences/Seminar/Workshop Attended in the Related topic /Inter Disciplinary National Label	05 Marks	Additional Marks	05 Marks
4.Conferences/Seminar/Workshop Attended in the Related topic /Inter Disciplinary Inter National Label	10 Marks	Additional Marks	10 Marks
5. Presented of your works with the Related topic / Inter Disciplinary in the UGC/DST/CSIR or other Recognized Institutions State/University Label Conferences/Seminar/Workshop	05 Marks	Additional Marks	05 Marks
6. Presented of your works with the Related topic /Inter Disciplinary in the UGC/DST/CSIR or other Recognized Institutions National Label Conferences/Seminar/Workshop	10 Marks	Additional Marks	10 Marks
7. Presented of your works with the Related topic /Inter Disciplinary in the UGC/DST/CSIR or other Recognized Institutions Inter National Label Conferences/Seminar/Workshop	15 Marks	Additional Marks	15 Marks
8. Related works Published in Recognized Refereed but not UGC Care Listed Journals	20 Marks	Additional Marks	20 Marks
9. Related works Published in Recognized Refereed UGC Care Listed /Scopus/SCI without Impact Factor Journals	25 Marks	Additional Marks	25 Marks
10. Related works Published in Recognized Refereed UGC Care Listed /Scopus/SCI with Impact Factor Journals	40 Marks	Additional Marks	40 Marks
11.Presentation of works (at least 15 minute)	30 Marks	-	30 Marks
12.Submission of Progress Report/ Thesis in hand written/Printed 20/30 or more pages.	30 Marks	-	30 Marks
13.Viva-Voice	30 Marks	-	40 Marks
14. Total(Maximum) Marks	100	-	100

17. **Attendance:** 80 % (Attendance to be verified by head/principal of university/college.)
Relaxation in minimum attendance requirement should be given only in the case of indoor hospitalization.

Record of class attendance: Each Instructor shall maintain a record of the student's attendance in each course thought by him in each semester

Minimum Class attendance: Each student shall be regular in attending classes and shall be required to have a minimum of 80% attendance in each course in each semester, failing which he/she shall not be awarded grade in that course, unless withdrawal from the course is permitted.

The percentage of attendance of a student in course in a semester shall be computed on the basis of the total number of lecture's, practical's and tutorial's attended by him/her and those actually held between the date of commencement of instruction and the date of closing instruction, respective of the date of him/her registration and/or the duration of leave duly granted to him/her.

The Vice Chancellor / Dean/ Principal may on the recommendation of the instructor/advisor concerned, through the Head of the Department, condone shortage in attendance up to 5% in a course(s) in exceptional circumstances and allow students with an attendance of 75% or more to appear at the final examination. However, on the recommendation of the Vice Chancellor/ Head of Institutions may grant a condonation to the extent of 5% and allow students with an attendance of 70% or more to appear at the final examination. In a very exceptional case, if a student fails to secure even 70 % attendance, his case can be referred to the Academic Council through Dean/Head of Institution for condonation to the extent of further 5 % and allow students with an attendance of 65 % or more.

Note:(i) In computation of percentage of attendance, fraction of 0.5 or more shall be counted as 1.
(ii) If student is called upon to repeat a course but he/she has already in required attendance in that course on a previous occasion, above requirements of attendance will not apply in his/her case.
(iii) Whenever students resort to mass absence from classes, a fine of Rs. 15.00/student/day may be levied from all such students. All such students will have to pay this fine before the final examination of the next semester and failure to do so shall render them liable to be debarred from appearing in the examination.

18. **Preparation of Mid-Term Examination Schedule:** The Mid-Term examination schedule shall be prepared and notified by the Principal/Head of Department of the college/university ten days before the commencement of the examination.
19. **Arrangements:** The Principal/ Head of Department of the college/university shall conduct the Mid-Term examination and the respective centre superintendents shall make the seating arrangements.
20. **Supply of Mid-Term Examination Material:**
(i) Examination materials such as question paper, answer books twine, drawing paper, log tables,

graph papers etc. will be supplied by the Principal/HoD/Centre Superintendent.

(ii) Every student shall be required to bring examination materials such as set squares, scales, pen, pencils, high litres etc. as he shall not be permitted to borrow any of these materials from fellow student in the examination hall.

21. **Unfair means in mid-term examination:** A student if found using unfair during mid-term examination, he will be awarded zero in mid-term examination.
22. **Make-Up (or missed) Mid-Term Examination:** Normally no make-up Mid-Term examination shall be permissible in lieu of the missed mid-term examination except as permitted by principal/HoD of College/University in extremely genuine cases, on the following grounds:
- If he/she is seriously ill.
 - If he/she has taken leave on account of the death of his mother, brother, sister, spouse, child or grandparent.
 - Any other genuine cause with which the HoD/principal is satisfied. Such cases should be reported to the Principal/Registrar in case of college/university.
 - Only one (additional) make-up(missed) examination will be permissible during a semester.
23. There shall be 500 marks each for Semesters. Thus for the entire course it comes out to be a total of 2000 marks.

24. Semester-VII are:

S.No.	Paper	Course Code	Paper Title	Nature of Paper	Credits	Marks
1.	I	B030701T	Algebra-I	Theory	4	100=25+75
2.	II	B030702T	Real Analysis	Theory	4	100=25+75
3.	III	B030703T	Point-Set Topology	Theory	4	100=25+75
4.	IV	B030704T	Partial Differential Equation and Calculus of Variations	Theory	4	100=25+75
5.	V	B030705P	Programming in C	Practical	4	100=25+75
6.	VI	Minor Elective (Other than own faculty)			4/5/6	Satisfactory or Not Satisfactory
7.	VII	B030706R	Topic and Supervisor Allotment for the Dissertation/Project Work/ Survey/ Industrial Training			

25. Semester-VIII are:

S.No.	Paper	Course Code	Paper Title	Nature of Paper	Credits	Marks
1.	I	B030801T	Complex Analysis	Theory	5	100=25+75
2.	II	B030802T	Classical Mechanics	Theory	5	100=25+75
3.	II	B030803T	Functional Analysis	Theory	5	100=25+75
4.	IV	B030804T	Elective Any one of the following	Theory	5	100=25+75
Fourier Analysis and Summability Theory Differential Geometry Extended MATLAB Special Theory of Relativity						
5.	V	B030805R	Evaluation of the Dissertation/ Project Work/ Survey/ Industrial Training by Progress report (in hand written/printed 20 or more pages), Presentation & Viva-Voice		8	100

26. Semester-IX are:

S.No.	Paper	Course Code	Paper Title	Nature of Paper	Credits	Marks
1.	I	B030901T	Algebra-II	Theory	4	100=25+75
2.	II	B030902T	Integral Equation and Boundary Value Problems	Theory	4	100=25+75
3.	III	B030903P	Wavelet	Practical	4	100=25+75
4.	IV	B030904T	Elective-I Any one of the following	Theory	4	100=25+75
Advanced Discrete Mathematics Advanced Linear Algebra Fluid Mechanics Probability Theory						
5.	V	B030905T	Elective-II Any one of the following	Theory	4	100=25+75
Advanced Ordinary Differential Equation Fuzzy Sets and Its Applications Non linear Analysis Special Function						
6.	VI	B030906R	Topic and Supervisor Allotment for the Project Work/ Survey/Industrial Training May be same as Previous Allotment			

27. Semester-X are:

S.No.	Paper	Course Code	Paper Title	Nature of Paper	Credits	Marks
1.	I	B031001T	Measure and Integration	Theory	5	100=25+75
2.	II	B031002T	Elective-I Any one of the following	Theory	5	100=25+75
Advanced Topology Algebraic Number Theory Mathematical Modelling Advanced Partial Differential Equations						
3.	III	B031003T	Elective-II Any one of the following	Theory	5	100=25+75
Advanced Special Function Mathematical Method Module Theory Wavelet Analysis						
4.	IV	B031004T	Elective-III Any one of the following	Theory	5	100=25+75
Differential Geometry of Manifolds Dynamical System General relativity and Cosmology Operator Theory						
5.	V	B031005R	Evaluation of the Project Work/Survey / Industrial Training by Presentation & Submission of Thesis in hand written/Printed 30 or more pages.		8	100

28. Detailed Syllabus for Semester-VII of P. G.(Mathematics):

B030701T Algebra-I:

Course Objectives: The paper of Algebra-I form is introduced to P.G. (Mathematics) Semester-VII classes for the study of structure of groups and properties of Arithmetic in Ring. The main objective of group theory and Division in ring is that to prepare the students for further research in modern algebra and Number theory. In this paper we introduced the Prominent Personalities: whose contributions in the field of mathematical, Social, Political Educational upliftment of india. The objective of this unit to developed the ethical and moral sense in the students.

Unit I

Prominent Personalities: whose contributions in the field of mathematical, Social, Political Edu-

cational upliftment of india. Ashutosh Mukherjee (1864-1924), Srinivasa Ramanujan (1887-1920), A. P. J. Abdul Kalam (1931-2015), Sarvepalli Radhakrishnan (1888-1975), Savitribai Phule (1831-1897), Pt. Madan Mohan Malaviya (1861-1946), Nation Father Mahatma Gandhi (1869-1948), Dr. B. R. Ambedkar (1891-1956).

Unit II

Group-Automorphisms, Inner automorphism, Automorphism groups and their computations. Conjugacy relation. Normaliser, Counting principle and the class equation of a finite group. Center for Group of prime-order. Abelianizing of a group and its universal property. Action of a group G on a set S , Examples, Stabilizer (Isotropy) subgroups and Orbit decomposition, Class equation, Translation and conjugation actions, Transitive and effective actions, Burnside theorem, Core of a subgroup.

Unit- III

Sylows theorems, p -groups, Direct Products of the Groups, Structure of groups of order pq , Characterization of finite Abelian groups and finite cyclic groups of specific orders in terms of Sylow subgroups. Normal series and composition series, Jordan-Holders theorem.,Commutator or derived subgroup, Solvable groups, Solvability of subgroups and factor groups and of finite p -groups, Examples, Lower and upper central series, Nilpotent groups.

Unit IV

Arithmetic In Ring: Division in Rings, Arithmetic and Ideals, Principal Ideal Domains (PID), Arithmetic in PID, Euclidean Domain (ED), Chinese Remainder Theorem in Rings, Unique factorization Domain(UFD), Gauss Theorem, primitive polynomial, Criteria for irreducibility of polynomials, Eiseintein irreducibility criteria.

Books Recommended:

1. I. N. Herstein, Topics in Algebra, Wiley Eastern, 1975.
2. P. B. Bhattacharya, S. K. Jain and S. R. Nagpal, Basic Abstract Algebra (2nd Edition), Cambridge University Press, Indian Edition 1977.
3. Ramji Lal, Algebra 1 and Algebra 2, Infosys Science foundation Series in Mathematical Sciences, Springer, Singapore, 2017.
4. D. S. Dummit and R.M. Foote, Abstract Algebra, John Wiley, N.Y., 2003.
5. T. W. Hungerford, Algebra, Springer (India) Pvt. Ltd., New Delhi, 2004.
6. J. B. Fraleigh, A first course in Abstract Algebra, Pearson Education, inc. 2002.

Course Outcomes: After the completion of the course, the student shall be able to

CO 1. Understand Group theory covering a wide area of research in abstract algebra.

CO 2. Understand Sylows theorems, group homomorphism, isomorphism etc are used to define the structure of groups as well as it is applicable in physical and chemical sciences and abstract concept of division in ring.

CO 3. Again conceptual understanding of the course for qualifying various competitive exams such as CSIR-NET (JRF), IAS, PCS and other teaching jobs.

B030702T Real Analysis

Course Objectives: The paper of Real Analysis is introduced to two year/four semester P.G. (Mathematics) classes for the study of functions of bounded variation, Riemann-Stieltjes integrals, point-wise convergence, uniform convergence and power series. The main objective of real analysis is that to prepare the students for further research in analysis and Functions of Several Variable.

Unit I

Countable and uncountable sets, Infinite sets and the axiom of choice, Cardinal numbers and its arithmetic, Schroeder-Bernstein theorem, Zorns Lemma, Well ordering principle, Functions of Bounded Variation and some properties of function of bounded variation, Lipschitz condition and function, Variation function, Positive Variation function, Negative Variation function and The Jordan Decomposition theorem.

Unit-II

Definition and existence of Riemann-Stieltjes integral. Properties of the integral, Integration and differentiation, the fundamental theorem of Calculus, integration of vector-valued functions. Rectifiable Curves.

Unit-III

Rearrangements of terms of a series, Riemann's theorem. Sequences and series of function, pointwise and uniform convergence. Cauchy criterion for uniform, convergence. Weierstrass M-test, Abel's and Dirichlet's tests for uniform convergence, uniform convergence and continuity, uniform convergence and Riemann-Stieltjes integration, uniform convergence and differentiation, Weierstrass approximation theorem, Power series, uniqueness theorem for power series, Abel's and Tauber's theorems.

Unit-IV

Functions of Several Variable, Linear transformation from \mathbb{R}^n into \mathbb{R}^m , Derivatives in an open subset of \mathbb{R}^n , Chain rule, Partial derivatives and total derivatives as a linear transformation, Derivatives in an open subset of higher orders, Taylor's theorem, Inverse function theorem, Implicit function theorem, Jacobians, extremum problems with constraints, Lagrange's multiplier method, Differentiation of integrals. Partitions of unity, Differential forms, Stoke's theorem.

Books Recommended :

1. Shanti Narayan, A Course of Mathematical Analysis, S. Chand & Co., New Delhi.
2. T.M. Apostol, Mathematical Analysis, Narosa Publishing House, New Delhi, 1985.
3. Walter Rudin, Principles of Mathematical Analysis, McGraw Hill Kogakusha, 1976.
4. E. Hewitt and K. Stromberg, Real and Abstract Analysis, Berlin, Springer, 1969.
5. Gabriel Klambauer, Mathematical Analysis, Marcel Dekkar, Inc., New York, 1975.
6. T.P. Natanson. Theory of Functions of Real Variable, Vol. I, Frederick Unger Publishing Co. 1961.
7. H. L. Royden: Real Analysis, Macmillan Pub. Co. Inc. New York, 4th Edition, 1993.

Course Outcomes: After the completion of the course, the student shall be able to

CO 1. Demonstrate an intuitive and computational understanding of functions of bounded variation, Riemann-Stieltjes integrals, point-wise convergence, uniform convergence and power series.

CO 2. Enter into wide area of research in analysis and Function of Several Variable.

CO 3. Get benefit of this course in various national and international competitive examinations.

B030703T Point-Set Topology:

Course Objective: A topology is a rubber sheet Geometry. Topology is concern with the properties of a geometric object that are preserved under continuous deformations such as stretching, twisting crumpling and bending but not tearing and gluing.

Unit I

Topological spaces, Closed sets, Open sets, Closure, Dense subsets, Neighbourhoods, exterior of a set, interior of a set, closure of a set, boundary of a set, Accumulation points and derived sets, Bases and subbases, Subspaces and relative topology. Separable space, Neighbourhood systems, first countable space, second countable space, Continuous functions and its characterizations via the closure and interior, open map, closed map, Homeomorphism, product of two spaces, quotient of a space, Compact space, Connected space, path connected space, components.

Unit II

Separation axioms T_0 -space, T_1 -space, T_2 -space, regular space, T_3 -space, completely regular space, $T_{3\frac{1}{2}}$ -space, normal space, T_4 space, their characterizations and basic properties. Urysohn's lemma, Tietze extension theorem, and its application and examples.

Unit III

Compactness, Continuous functions and compact sets, Basic properties of compactness. Compactness and finite intersection property. Sequentially and countably compact sets. Local compactness and one point compactification, Compactness in metric spaces, Equivalence of compactness, countable compactness and sequential compactness in metric spaces. Connected spaces. Connectedness on the real line. Components. Locally connected spaces.

Unit IV

Nets and filters, Topology and convergence of nets. Hausdorffness and nets, Filters and their convergence, Ultra filters, Canonical way of converting nets to filters and vice-versa, Tychonoff Theorem (Statements and Sketch of proofs).

Books Recommended:

1. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill, 1963.
2. J. R. Munkres, Topology, Narosa Publishing House, New Delhi, 2005.
3. K. D. Joshi, Introduction to General Topology, Wiley Eastern, 1983
4. S.W. Davis Topology, Tata McGraw Hill, 2006
5. Sze-Tsen Hu, Elements of General Topology, Holden-Day Inc., 1964.

Further Reading:

1. N. Bourbaki, General Topology, Part I, Addison-Wesley, 1966.
2. J. Dugundji, Topology, Prentice-Hall of India, 1966.

Course Outcome: After the completion of the course, the student shall be able to

CO 1. Apply Knot theory a branch of topology is used in biology to study the effects of certain enzymes on DNA.

CO 2. Understand the application of Topology is relevant to physics in areas such as condensed matter, quantum field theory, differential geometry of manifolds & physical cosmology.

CO 3. Gain conceptual understanding of the course for qualifying various competitive exams such as CSIR- NET(JRF), IAS, PCS and other teaching jobs.

B030704T Partial Differential Equation and Calculus of Variations:

Course Objective: The paper of Partial Differential Equation and Calculus of Variations is introduced to P.G.(Mathematics) classes for the study of basic definition of Partial differential equation and Calculus of Variations classification and geometrical Interpretation. The main objective of Partial Differential Equations is that to prepare the students for further research in applied mathematics.

Unit-I:

Formation of P.D.Es, First order P.D.E.s, Classification of first order P.D.E.s, Complete, general and singular integrals, Lagranges or quasi-linear equations, Integral surfaces through a given curve, Orthogonal surfaces to a given system of surfaces, Characteristic curves.

Unit-II:

Pfaffian differential equations, Compatible systems, Charpits method, Jacobis Method. Linear equations with constant coefficients, Reduction to canonical forms, Classification of second order P.D.E.s and its solutions

Unit-III

Calculus of Variational Problems with Fixed Boundaries: Introduction, Functional, Closeness of Functions, Maximum and Minimum Values of a Functional, Euler's Equation for Functionals, Extremals, Functional Dependent on Higher order Derivatives and more than one variable, Variational Problems in Parametric Form, Invariance of Euler's Equation under coordinate Transformation, Conditional Extremum.

Unit-IV

Calculus of Variation-Variational Problem with Moving Boundaries: Functionals dependent on one and two functions. One sided variations. Sufficient conditions for an Extremum-Jacobi and Legendre conditions. Second Variation. Variational Principle of least action.

Books recommended:

1. I.N. Sneddon: Elements of Partial Differential Equations, McGraw-Hill Pub.,1957.
2. T. Amaranath: An Elementary Course in Partial Differential Equations, Narosa Pub. 2005.
3. A.S. Gupta, Calculus of Variations with Applications, Prentice Hall of India, 1997.
4. I.M. Gelfand and S.V. Fomin, Calculus of Variations, Prentice Hall.

Course Outcome: After the completion of the course, the student shall be able to

CO 1. Describe real- world system using partial differential equations and Calculus of Variations.

CO 2. Solve Calculus of Variation-Variational Problem with fixed and Moving Boundaries, first order partial differential equations and second order partial differential equations using different methods.

CO 3. Determine the existence and uniqueness conditions of solutions of Partial differential equations, Calculus of Variation-Variational Problem with fixed and Moving Boundaries.

B030705P: Programming in C

Course Objective: The paper of Programming in C is introduced to P.G.(Mathematics) classes for the study of basic definition and Programmers model of computer, Algorithms. Also use, in the real word Problems. The main objective of Programmers model of computer, Algorithms is that to prepare the students for further research in computational mathematics.

Unit-I:

Programmers model of computer, Algorithms, Flow Charts, Constants, Variables, Data types, Operators and expression, formatted input and output, Decision makings, Branching and Looping, Arrays, User defined functions, Structures, Pointers.

Unit-II

Simple Programming based on Above theory, Programming to find the root of Transcendental and algebraic Equations.

Unit-III

Programming to find the integral using Numerical methods, Programming for Interpolation

Unit-IV

Programming for addition, subtraction and multiplication of matrices upto order 4×4 .

Recommended Books:

1. V. Rajaraman, Programming in C, Prentice Hall of India, 1994.
2. E. Balagurusamy, Programming in ANSI C, Tata McGraw Hill New Delhi.
3. M. K. Jain, S. R. K. Iyengar, R. K. Jain, Numerical Methods for Scientific and Engineering Computation. New Age International (P) Ltd. 1999.
4. M. K. Jain, S. R. K. Iyengar, R. K. Jain, Numerical Methods Problems and Solutions. New Age International (P) Ltd. 1996.

Further Reading:

1. V. Rajaraman, Programming in C, Prentice Hall of India, 1994.
2. E. Balagurusamy, Programming in ANSI C, Tata McGraw Hill New Delhi.
3. M. K. Jain, S. R. K. Iyengar, R. K. Jain, Numerical Methods for Scientific and Engineering Computation. New Age International (P) Ltd. 1999.
4. M. K. Jain, S. R. K. Iyengar, R. K. Jain, Numerical Methods Problems and Solutions. New Age

International (P) Ltd. 1996.

Course Outcome: After the completion of the course, the student shall be able to

CO 1. Describe real- world system Programmers model of computer for example artificial intelligence etc.

CO 2. Solve the integral using Numerical methods, Programming for algebraic, transcendental, differentiation, integral and Interpolation.

CO 3. Determine Algorithm and Programmes for the real word Problems.

CO 4. Use this as a tool of computational research.

29. Detailed Syllabus for Semester-VIII of P.G.(Mathematics):

B030801T Complex Analysis:

Course Objective: The paper of Complex Analysis is introduced to two year or four semester P.G. (Mathematics) classes for the study of improper integrals by contour integration method, power series and its region of convergence, analytic continuation, conformal mapping, Schwarz's lemma and related results for further study. The main objective of complex analysis is that to prepare the students for further research in analysis and complex analysis.

Unit I

Evaluation of definite and improper integrals by contour integration method, Conformal Mapping, Mobius (Bilinear) transformations: involving circles and half-planes, fixed point, cross ratio, Transformations $w = z^2$, $w = \tan^2(z/2)$, Univalent function and its properties. Many valued functions and its properties.

Unit II

Power series and its convergence. Analyticity of power series, singularity of power series, Maximum-modulus theorem. Schwarz's lemma. Hadamard's three-circles theorem. Borel- Cartheodory theorem. Phragmen- Lindelof theorem.

Unit III

Analytic continuation. Uniqueness of analytic continuation. Power series method of analytic continuation. Natural boundary.

Unit IV

The Gamma function $\Gamma(z)$, Definition, Analytic Character of $\Gamma(z)$, Euler's integrals, Tannery's theorem, Euler's limit formula for $\Gamma(z)$, two identities, Legendre's duplication formula, Euler's constant, Canonical product for $\Gamma(z)$, Zeta Function.

Books Recommended:

1. E.C. Titchmarsh: Theory of Functions, Oxford University Press, London.

2. Mark J. Ablowitz and A.S. Fokas: Complex Variables: Introduction and Applications, Cambridge University Press, South Asian Edition, 1998.
3. R.V. Churchill & J.W. Brown. Complex Variables and Applications, 5th Edition McGraw-Hill, New York, 1990.
4. Shanti Narayan: Theory of Functions of a Complex Variable, S. Chand & Co., New Delhi.
5. S. Ponnusamy, Foundation of Complex Analysis, Narosa Publication

Course Outcomes: After the completion of the course, the students shall be able to

CO 1. Understand the use of this course in different field of mathematical Analysis.

CO 2. Think and develop new ideas in complex analysis.

CO 3. Get benefit of this course in various national and international competitive examinations like UGC-CSIR NET (JRF) etc.

B030802T Classical Mechanics

Course Objectives: The objective of this paper is to study generalized coordinates, Holonomic and non holonomic systems, Lagrangian and Hamiltonian approach, small oscillations and canonical transformations. The main objective of this paper is that to prepare the students for further research in Classical Mechanics and applied mathematics.

Unit I

The momentum of a system of particles, the linear and the angular momentum, rate of change of momentum and the equations of motion for a system of particles, principles of linear and angular momentum, motion of the center of mass of a system, theorems on the rate of change of angular momentum about different points, with special reference to the center of mass, the kinetic energy of a system of particles in terms of the motion relative to the center of mass of the system.

Unit II

The angular momentum and kinetic energy of a rigid body in terms of inertia constants, Equations of motion. Eulers dynamical equations of motion, Eulers geometrical equations of motion, Motion under no forces, the invariable line and the invariable cone, Instantaneous axis of rotation, Eulerian angles

Unit III

Generalized co-ordinates, geometrical equations, holonomic and non-holonomic systems, configuration Space, Lagranges equations using D Alemberts Principle for a holonomic conservative system, Lagrangian function, deduction of equation of energy when the geometrical equations do not contain time t explicitly, Lagranges multipliers case, deduction of Eulers dynamical equations from Lagranges equations, Lagrange equations for impulsive motion.

Unit IV

Generalized momentum and the Hamiltonian for a dynamical system, Hamiltons canonical equations of motion, Hamiltonian as a sum of kinetic and potential energies, phase space and Hamiltons

Variational principle, Hamilton's principle function, the principle of least action, canonical transformations, conditions of canonicity, Hamilton-Jacobi (H-J) equation of motion (outline only), Poisson-Brackets, Poisson-Jacobi identity, Poisson's first theorem.

Motivating problems of calculus of variations, Shortest distance, Minimum surface of evaluation, Brachistochrone problem, Isoperimetrical problem

Books Recommended:

1. E. A. Milne, Vectorial Mechanics, Methuen & Co. Ltd., London, 1965.
2. A. S. Ramsey, Dynamics, Part II, CBS Publishers & Distributors, Delhi, 1985.
3. H. Goldstein, Classical Mechanics, Addison-Wesley Publishing Company, London, 1969.
4. N. Kumar, Generalized Motion of Rigid Body, Narosa Publishing House, New Delhi, 2004.

Course Outcomes: After the completion of the course, the student shall be able to

CO 1. Understand the Lagrangian and Hamiltonian formalisms of the laws of motion, which are generalizations of Newton's equations of motion.

CO2. Apply the formalisms of analytical dynamics to practical examples of small oscillations.

CO 3. Apply the related mathematical methods such as coordinate transformations and variational methods to study analytical dynamics to real life problems.

B030803T Functional Analysis:

Course Objective: Banach spaces originally grew out of the study of function spaces. The theory of Banach spaces developed in parallel with the general theory of linear topological spaces. These theories mutually enriched one another with new ideas and facts. A Banach space is a vector space with a metric that allows the computation of a vector length and distance between vectors and is complete in the sense that Cauchy sequence of a vectors always converges to a well-defined limit that is within the space. Hilbert space is an abstract vector space possessing the structure of an inner product that allows length and angle to be measured. The main objective of this paper is that to prepare the students for further research in functional analysis.

Unit I

Norm and its properties, Normed linear spaces, Banach spaces, the sequence spaces and the function spaces as Banach spaces, Characterization of Continuous linear transformations between two normed spaces, Bounded linear operators, $B(X, Y)$ as a normed linear space.

Unit II

Hahn-Banach Theorem, Open mapping theorem, Closed graph theorem, Banach-Steinhaus theorem, Uniform boundedness principle, Conjugate spaces, Weak and Weak*-topology on a conjugate space, Simple Application to reflexive separable spaces and to the Calculus of Variation.

Unit III

Hilbert Spaces, Schwarz inequality, orthogonal complement of a subspace, orthonormal bases, Continuous linear functionals on Hilbert spaces, Riesz Representation Theorem, Reflexivity of Hilbert

Spaces, Applications of polarization identity.

Unit IV

The adjoint of an operator, Self adjoint operators, Normal and unitary operators, Projections, Compact operator and its simple properties, Finite dimensional spectral theory Spectrum of an operator, the Spectral theorem.

Books Recommended:

1. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill, 1963.
2. S. Ponnusamy, Foundations of Functional Analysis, Narosa Publishing House, New Delhi, 2002.
3. G. Bachman and L. Narici, Functional Analysis, Academic Press, 1966.
4. A. E. Taylor, Introduction to Functional Analysis, John Wiley, 1958.
5. N. Dunford and J. T. Schwartz, Linear Operators, Part-I, Interscience, 1958.
6. R. E. Edwards, Functional Analysis, Holt Rinehart and Winston, 1965.
7. C. Goffman and G. Pedrick, First Course in Functional Analysis, Prentice- Hall of India, 1987.

Course Outcomes: After the completion of the course, the student shall be able to

CO 1. Know basics of Banach space as a part of functional analysis.

CO 2. Determine the significance of functional analysis modern applied and computational mathematics.

CO 3. Understand the application of Hilbert spaces in Mathematics and Physics typically as infinite dimensional function space.

CO 4. Apply Hilbert spaces in solving partial differential equations and in studying Fourier analysis, etc.

CO 5. Get benefit of this course in various national and international competitive examinations

B030804T Elective

Course Objectives: The objective of these papers are to know different method to solve the problems of Fourier Analysis and Summability Theory in advanced level, Differential Geometry, Extended MATLAB, General Relativity. This theory can be used to solve many rigorous problems of universe. The main objective of fourier analysis and summability theory is that to prepare the students for further research in summability theory and analysis.

Fourier Analysis and Summability Theory:

Unit I

Convergence problem, Dirichlet's integral, Riemann-Lebesgue Theorem, Convergence tests, Dini's, Jordan's, de la Vallee-Poussin's tests and their inter-relations. Summation of series by arithmetic

means

Unit-II

Summability of Fourier series, Fejer's theorem, Weierstrass's approximation theorem, Almost everywhere summability, The Fejer-Lebesgue theorem, A continuous function with a divergent Fourier series, Order of partial sums, Integration of Fourier series, Convergent trigonometric series need not be a Fourier series, Parseval's theorem.

Unit III

Functions of the class L^2 : Bessel's inequality, Parseval's theorem for continuous functions, The Riesz- Fischer theorem, Properties of Fourier coefficients, Uniqueness of trigonometric series, Cantor's lemma, Riemann's First and second theorems.

Unit-IV

Special methods of summation: Norlund means, Regularity and Consistency of Norlund means, Inclusion, Equivalence, Euler's means, Abelian means, Riesz's typical means. Arithmetic means: Holder's means, simple theorems concerning Holder summability, Cesaro means, means of non-integral orders, simple theorems concerning Cesaro summability, Cesaro and Abel summability, Cesaro means as Norlund means, Tauberian theorems for Cesaro summability.

Recommended Books:

1. E.C. Titchmarsh: A Theory of Functions, Oxford University Press, 1939.
2. A Zygmund: Trigonometric series Vol. I, The University Press, Cambridge 1959
3. G. H. Hardy: Divergent series, The Clarendon Press, Oxford, 1949.

Differential Geometry:

Unit I

Curves in space \mathbb{R}^3 , Parameterization of curves, Regular curves, Helices, Arc length, reparameterization (by arc length), tangent, principal normal, binormal, osculating plane, normal plane, rectifying plane, curvature and torsion of smooth curves, Frenet- Serret formulae, Frenet approximation of a space curve.

Unit II

Osculating circle, osculating sphere, spherical indicatrices, involutes and evolutes, intrinsic equations of space curves, isometries of \mathbb{R}^3 , fundamental theorem of space curves, surfaces in \mathbb{R}^3 , regular surfaces, co-ordinate neighborhoods, parameterized surfaces, change of parameters, level sets of smooth functions on \mathbb{R}^3 , surfaces of revolution, tangent vectors, tangent plane, differential of a map. Normal fields and orientability of surfaces, angle between two intersecting curves on a surface, Gauss map and its properties, Weingarten map, second and third fundamental forms, classification of points on a surface.

Unit III

Curvature of curves on surfaces, normal curvature, Meusnier theorem, principal curvatures, geometric interpretation of principal curvatures, Euler theorem, mean curvature, lines of curvature, umbilical points, minimal surfaces, definition and examples, Gaussian curvature, intrinsic formulae for the Gaussian curvature, isometries of surfaces, Gauss Theorem Egregium (statement only).

Unit IV

Christoffel symbols, Gauss formulae, Weingarten formulae, Gauss equations, Codazzi-Mainardi equations, curvature tensor, geodesics, geodesics on a surface of revolution, geodesic curvature of a curve, Gauss-Bonnet Theorem (statement only).

Books Recommended:

1. D. Somasundaram, Differential Geometry, A First Course, Narosa Publishing House, New Delhi, 2005.
2. M. P. Do Carmo, Differential Geometry of Curves and Surfaces, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1976.
3. J. A. Thorpe, Elementary Topics in Differential Geometry, Springer (Undergraduate Texts in Mathematics), 1979.
4. B. O'Neill, Elementary Differential Geometry, Academic Press, 1997.
5. A. Pressley, Elementary Differential Geometry, Springer (Undergraduate Mathematics Series), 2001.

Extended MATLAB :

Unit-I:

Mathematical built-in functions, Creating the different types of arrays and matrices, handling the element/row/column of an array/matrix. Programming structure, Script files, Functions files, Debugging programs.

Unit-II:

Loops, branches and control flow, Relational and logical operations. MATLAB graphics: Two and three dimensional graphics, Multiple plots, Plot properties.

Unit-III:

Creating symbolic variables, array and matrices. Differentiation, Integration, simplification, transform and equation solving.

Unit-IV:

Implementation of functions for numerical solutions to the differential equations/system of differential equation and integration problems and problems of optimization.

Suggested Problems for Practical:

1. Problems of Optimization Theory
2. Problems of Numerical Analysis
3. Mathematical Modeling problems based on differentiation and integration

References:

1. Text books (optional): MATLAB: A Practical Introduction to Programming and Problem Solv-

ing. Attaway, Stormy. 2012. Full text online using OSU online library.

2. Notes and supplementary lecture slides.

3. Matlab online glossary: <http://people.sc.fsu.edu/~jburkardt/html/matlab-glossary.html> 5) Matlab Central: <http://www.mathworks.com/matlabcentral/>

Special theory of relativity:

Unit-I

Review of Newtonian mechanics- Inertial frames. Speed of light and Galilean relativity. Michelson-Morley experiment. Lorentz-Fitzgerald contraction hypothesis. Relative character of space and time. Postulates of special theory of relativity. Lorentz transformation equations and its geometrical interpretation.

Unit-II

Group properties of Lorentz transformations. Relativistic Kinematics- Composition of parallel velocities. Length contraction. Time dilation. Transformation equations for components of velocity and acceleration of a particle and Lorentz contraction factor.

Unit-III

Geometrical representation of space-time-Four dimensional Minkowskian space-time of special relativity. Time like, light like and space like intervals. Null cone, Proper time. World line of a particle. Four vectors and tensors in Minkowskian space-time. Relativistic mechanics- Variation of mass with velocity.

Unit-IV

Equivalence of mass and energy. Transformation equations for mass momentum and energy. Energy-momentum for vector. Relativistic force and Transformation equations for its components. Relativistic equations of motion of a particle. Energy momentum tensor of a continuous material distribution. Electromagnetism- Maxwell's equations in vacuo.

Transformation equation for the densities of electric charge and current. Propagation of electric and magnetic field strengths. Transformation equations for electromagnetic four potential vector. Transformation equations for electric and magnetic field strengths. Gauge transformation. Lorentz invariance of Maxwell's equations.

Books Recommended :

1. C. Moller, The Theory of Relativity, Oxford Clarendon Press, 1952.
2. J.L. Anderson, Principles of Relativity Physics, Academic Press, 1967.
3. W. Rindler, Essential Relativity, Van Nostrand Reinhold Company, 1969.
4. R. Resnick, Introduction to Special Relativity, Wiley Eastern Pvt. Ltd. 1972.

Course Outcomes: After the completion of these courses, the student shall be able to

CO 1. Understand the basics of Fourier series and summability theory.

CO 2. Understand the basic of Differential Geometry and demonstrate an intuitive and computational understanding of Tensor Algebra, Differentiable manifold, Riemannian Manifold, Exterior algebra and Submanifolds & Hyper surfaces.

CO 3. Use the knowledge of MATLAB in different computational analysis and research.

CO 4. Understand the beauty of general Relativity theory as a bridge between physics of Universe with their geometry.

CO 5. Apply general relativity to describe the evolution of universe and most of the cosmic problems.

CO 6. Know the importance of this theory in solving the problem of universe with the differential geometry and geometric structures.

30. Detailed Syllabus for Semester-IX of P.G. (Mathematics):

B030901T Algebra-II:

Course Objective: The paper of Algebra-II is introduced to two year/ four semester P.G.(Mathematics) classes for the study of extension field and related results, algebraic and transcendental extension, splitting fields, normal extensions, perfect field, finite fields, Galois group, modules, cyclic modules and related results. The main objective of this paper is that to prepare the students for further research in analysis and modern algebra.

Unit I

Field extensions, Degree of extension, Finite extensions, Algebraic and transcendental elements, Algebraic and transcendental extensions, Simple extensions, Primitive element of the extension, Splitting fields and their uniqueness, Normal extensions, Separable extensions, Perfect fields, Transitivity of separability, Algebraically closed field and Algebraic Closure, Normal closures, Dedekind's theorem.

Unit II

Automorphisms of fields, K -automorphisms, Fixed fields, Galois group of the extension field, Abelian extension, Cyclic extension, Galois extensions, Fundamental theorem of Galois theory, Computation of Galois groups of polynomials.

Unit III

Finite fields, Existence and uniqueness, Subfields of finite fields, Characterization of cyclic Galois groups of finite extensions of finite fields, Solvability by radicals, Galois characterization of such solvability, Generic polynomials, Abel-Ruffini theorem, Geometrical constructions. Cyclotomic extensions, Cyclotomic polynomials and its computations, Cyclotomic extensions of \mathbb{Q} , Galois groups of splitting fields of $x^n - 1$ over \mathbb{Q} .

Unit IV

Modules, Submodules, Quotient modules. Homomorphism and Isomorphism theorems. Cyclic modules, Simple modules. Semi-simple modules. Schuler's lemma, Free modules, Noetherian and artinian modules and rings-Hilbert basis theorem, Wedderburn-Artin theorem (only Statement), Uniform modules, primary modules and Noether Lasker theorem (only Statement).

Books Recommended:

1. T. W. Hungerford, Algebra, Springer (India) Pvt. Ltd., New Delhi, 2004.
2. I. A. Adamson, An Introduction to Field Theory. Oliver & Boyd, Edinburgh, 1964.
3. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley, N.Y., 2003.
4. N. S. Gopalakrishnan, University Algebra, Wiley Eastern Ltd., New Delhi, 1986.
5. F. W. Anderson and K. R. Fuller, Rings and Categories of Modules, Springer, New York, 1974.
6. M. Artin, Algebra, Prentice-Hall of India, 1991.

Course Outcomes: After the completion of the course the student shall be able to

CO 1. Think and develop new ideas in this paper.

CO 2. Understand the applications of this course in different field of Science and Technology.

CO 3. Get benefit of this course in various national and international competitive examinations like UGC-CSIR NET-JRF, GATE, etc.

B030902T Integral Equation and Boundary Value Problems :

Course Objectives: The objective of course is to know different method to solve ordinary Boundary Value Problems and also to solve integral equations of Fredholm and Volterra type.

Unit-I

Classification of integral equations of Volterra and Fredholm types; Conversion of initial and boundary value problem into integral equation; Conversion of integral equation into differential equation (When it is possible)

Unit-II

Volterra and Fredholm integral operators and their iterated kernels; Resolvent kernels and Neumann series method for solution of integral equations; Branch contraction principle, its application in solving integral equations of second kinds by the method of successive iteration and basic existence theorem

Unit-III

Abel integral equation and tautochrone problem; Fredholm-alternative for Fredholm integral equation of second kind with degenerated kernels; Use of Laplace and Fourier Transform to solve integral equations.

Unit-IV

Definition of a boundary value problem for an ordinary differential equation of the second order and its reduction to a Fredholm integral equation of the second kind; Dirac Delta Function; Green Function for ordinary differential initial and boundary value problem.

Books Recommended :

1. R.P. Kanwai, Linear Integral Equation : Theory and Techniques. Academic Press, New York, 1971
2. S.G. Mikhlin, Linear Integral Equation. Hindustan Book Agency, 1960.

3. I.N. Sneddon. Mixed Boundary Value Problem in Potential Theory. North HOLLAND, 1966.

Course Outcomes: After the completion of the course, the students shall be able to

CO 1. Solving a wide range of Integral equations:

CO 2. Formulate and solve initial and boundary value problems.

CO 3. Solve linear Volterra and Fredholm integral equations using appropriate methods and understand the relationship between integral and differential equations and transform one type into another.

B030903P Wavelet:

Course Objectives: The objective of this paper is to study Introduction and Definition of Wavelets, Construction of wavelets on \mathbb{Z}_N , First stage and by iteration, The Haar system, Shannon wavelets, Daubechies D6 wavelets on \mathbb{Z}_N . Using this wavelets to compute numerical values of differential and integral problems involved many of the Physical problems and use it in image processing and area of Medical Sciences etc.

Unit I

Introduction and Definition of Wavelets, Construction of wavelets on \mathbb{Z}_N , First stage and by iteration, The Haar system, Shannon wavelets, Daubechies D6 wavelets on \mathbb{Z}_N . Description of $l^2(\mathbb{Z})$, $L^2[\pi, \pi)$, $L^2(\mathbb{R})$, their orthonormal bases, Fourier transform and convolution on $l^2(\mathbb{Z})$, wavelets on $l^2(\mathbb{Z})$, Haar wavelets on $l^2(\mathbb{Z})$, Daubechies D6 wavelets for $l^2(\mathbb{Z})$. Orthonormal bases generated by a single function in $L^2(\mathbb{R})$ using MATLAB.

Unit II

To plot a member in $l^2(\mathbb{Z}_N)$, its Fourier transform and its inverse Fourier transform using MATLAB.

Unit III

Computing Fourier coefficients of an element of $l^2(\mathbb{Z}_N)$, with respect to a given wavelet (Haar, Daubechie's D6) at a certain level such as $\langle z, \psi_{-2,k} \rangle$ etc. using MATLAB

Unit IV

Construct the Legendre Wavelets, Chebyshev Wavelets and Pseudo Chebyshev Wavelets, its application in Computing the approximation of functions and determine the Operation Matrix of Integration (OMI), PMO, Using this solve the problems of ODE/PDE and the Problem of the linear Integral Equation using MATLAB.

Books Recommended:

1. Ingrid Daubechies, Ten Lectures on Wavelets, CBS-NFS Regional Conferences.
2. Michael W. Frazier, An Introduction to Wavelets through Linear Algebra, Springer-Verlag, 1999.
3. C. K. Chui, An Introduction to Wavelets, Academic Press, 1992.
4. Eugenio Hernandez and Guido Weiss, A First Course on Wavelets, CRC Press, 1996.

Course Outcomes: After the completion of the course, the student shall be able to

CO 1. Understand the basics of this course with practical.

CO 2. Think and develop new ideas in this field.

CO 3. Understand the use of this course in science and technology and other fields of Mathematical Analysis for the practical purpose.

B030904T Elective-I

Course Objective: The objective of these papers are to study of Advanced Discrete Mathematics, Advanced Linear Algebra, Fluid Mechanics and Probability Theory. Advanced Discrete Mathematics is the study of mathematical structure that are countable otherwise distinct and separable. Concepts of discrete mathematics are useful in studying and describing abstract and problems in computer science. In the Advanced Linear Algebra is to study of Geometry and solution of system of equations and to learn the basic of Fluid Mechanics and Probability Theory.

Any one of the following

Advanced Discrete Mathematics:

Unit I

Mathematical Logic, Statement calculus: Propositional logic, Logic operators or connectives, Well formed formula (wff), Construction of truth-table for a formula, Equivalence of formulas, Tautology, Contradiction argument, Valid argument, Proving validity by truth-table methods, Inference theory of statement calculus, Minimal sets of logic operators. Predicate calculus: Statement function and statement, Proving validity by the deduction method, Inference rules, Proving validity by the method of contradiction.

Unit II

Lattice theory and Boolean algebra Lattice Theory: partial order relation, Partially ordered set, Totally ordered set, Hasse Diagrams, Lattice, Lattice as an algebraic system, Bounded lattice, Complemented lattice, Distributive lattice, Direct product, Lattice homomorphism. Boolean algebra: Boolean functions, Principle of duality, Boolean function minimization, Sum of products and product of sums form, Normal forms, Conversion of normal forms into principal normal forms, Boolean function minimization, Logic circuits, Designing logic circuits.

Unit III

Automata theory, Finite state automaton, Types of automaton, Deterministic finite state automaton, Non deterministic finite state automaton, Non deterministic finite-state automaton with ϵ , Equivalence of NFA and DFA, Equivalence of NFA and NFA-, Equivalence of NFA- and DFA, Finite state Machines: Moore and Mealy machine, and their conversion, Turing machine.

Unit IV

Grammars and Languages, Regular language, Regular expression Equivalence of Regular language

and finite state automaton, Grammar: Context-free and Context-sensitive grammar, LR Grammar: Construction of LR(0) parsing table, Construction of LR(1) parsing table, Decision algorithms for CFL.

Books Recommended:

1. John E. Hopcroft, Rajeev Motwani, Jeffrey D. Ullman: Introduction to Automata Theory, Languages and Computation, Pearson Education, 2000.
2. Mendelson, Elliott: Introduction to Mathematical Logic, Chapman & Hall, 1997.
3. Arnold B. H.: Logic and Boolean Algebra, Prentice Hall, 1962.
4. K. H. Rosen: Discrete Mathematics and its applications, MGH 1999.

Advanced Linear Algebra :

Unit I

Algebraic and geometric multiplicities of eigenvalues, Invariant subspaces, Annihilating polynomials, Minimal polynomials of linear operators and matrices, Characterization of diagonalizability in terms of multiplicities and also in terms of the minimal polynomial, Matrix limits and Markov chain.

Unit II

Invariant subspaces, Triangulability, Simultaneous triangulation and simultaneous diagonalization. Direct-Sum decomposition, Invariant direct sum, The Primary decomposition Theorem.

Unit- III

Inner Product Space, The Gram-Schmidt Orthogonalization process. and Orthogonal complements, The adjoint of a linear operator, Normal, Self-Adjoint and Unitary Operator and their matrices, Orthogonal projection and the spectral theorem, Singular value decomposition theorem and their Pseudo-inverse, Bilinear and Quadratic forms.

Unit- IV

Diagonalizable and nilpotent parts of a linear operator, Rational canonical form of matrices, Elementary Jordan matrices, Reduction to Jordan canonical form, Semisimple operators, Taylor formula. Positive definite matrices and polar decomposition, QR, LU and Cholesky decompositions of matrices, Singular value decomposition.

Books Recommended:

1. K. Hofmann and R. Kunze, Linear Algebra. Prentice Hall of India, New Delhi, 1972.
2. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley & Sons, N.Y., 2003.
3. H. Helson, Linear Algebra, Hindustan Book Agency, New Delhi, 1994.
4. N. Jacobson, Basic Algebra, Vol. 1, Hindustan Publishing Co., New Delhi, 1984.

Further Reading:

1. N. S. Gopalakrishnan, University Algebra, Wiley Eastern, New Delhi, 1986.
2. T. W. Hungerford, Algebra, Springer (India), Pvt. Ltd., 2004.
3. C. Musili, Rings and Modules, Narosa Publishing House, New Delhi, 1994.

Fluid Mechanics

Unit I

Relation between surface and volume integrals, Relation between line and surface integrals, Continuity equation in Cartesian, polar, spherical, cylindrical and generalized orthogonal curvilinear co-ordinate system, real and ideal fluids, Newtonian, Non-Newtonian fluids, some important types of flows

Unit II

Methods of fluid motion, material derivative, local derivative, convective derivative, significance of equation of continuity, equation of continuity by Euler's and Lagrangian method, some symmetrical forms of equation of continuity, stream line, path line, streak line, velocity potential or velocity function, vortex line, rotational and irrotational motion, equation of motion of inviscid fluid.

Unit III

One dimensional inviscid incompressible flow, Bernoulli's equation, pressure equation, Motion in two dimension, Stream function, Irrotational motion in two dimensions, Complex potential, Cauchy Riemann equations in polar form, Source and sink, Doublet, Milne-Thomson Circle Theorem, Complex potential for a uniform flow past a circular cylinder, Streaming and circulation about a fixed circular cylinder, Blasius Theorem, Conformal transformation: Uniform line distributions (source, vortex and doublet) under conformal transformation.

Unit IV

Body forces and surface forces, Nature of stresses, Transformation of stress components, Stress invariants, Principal stresses, Nature of strains, Rates of strain components, Relation between stress and rate of strain components, General displacement of a fluid element, Newtons law of viscosity, Navier-Stokes equation (sketch of proof).

Books Recommended:

1. A. S. Ramsey, A Treatise on Hydrodynamics, Part I, G. Bell and Sons Ltd. 1960.
2. L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Butterworth-Heinemann, 2nd Edition, 1987.
3. N. Curle and H. J. Davies, Modern Fluid Dynamics, Vol. I, D. van Nostrand Comp. Ltd., London, 1968.
4. S. W. Yuan, Foundations of Fluid Mechanics, Prentice-Hall, Englewood Cliffs, NJ, 1967.
5. F. Chorlton, Textbook of Fluid Dynamics, CBS Publishers, New Delhi, 2004.
6. G.K. Batchelor, An Introduction to Fluid Dynamics, Cambridge University Press, Cambridge Mathematical Library, 1967.

Probability Theory:

Unit-I:

Notion of probability: Random experiment, sample space, axiom of probability, elementary properties of probability, equally likely outcome problems. Random Variables: Concept, cumulative distribution function, discrete and continuous random variables, expectations, mean, variance, moment generating function.

Unit-II:

Discrete random variables: Bernoulli random variable, binomial random variable, geometric random variable, Poisson random variable. Continuous random variables: Uniform random variable, exponential random variable, Gamma random variable, normal random variable.

Unit-III:

Conditional probability and conditional expectations, Bayes theorem, independence, computing ex-

pectation by conditioning.

Unit-IV:

Bivariate random variables: Joint distribution, joint and conditional distributions, the correlation coefficient. Functions of random variables: Sum of random variables, the law of large numbers and central limit theorem, the approximation of distributions.

Books Recommended:

1. J.Jacod and P.Protter, Probability Essentials, Springer, 2004.
2. V.K. Rohatgi and A. K. Md. E.Saleh, An Introduction to Probability and Statistics, 2nd Edn.. Wiley, 2001.
3. P.G.Hoel, S.C.Port and C.J.Stone, Introduction to Probability Theory, Universal Book Stall, 2000.
4. G.R. Grimmett and D.R.Stirzaker, Probability and Random Processes, 3rd Edn.. Oxford University Press, 2001.

Further Reading

1. S.Ross, A First Course in Probability, 6th Edn. Pearson, 2002.
2. W. Feller, An Introduction to Probability Theory and its Applications, Vol.1, 3rd Edn. Wiley, 1968.
3. J.Rosenthal, A First Look at Rigorous Probability Theory, 2nd Edn.. World Scientific, 2006.

Course Outcomes: After the completion of these courses, the student shall be able to

CO 1. Study the significance of discrete mathematics as a modern trend in world-wise era of computer science especially Cryptography, Rational Databases, Logistics Computer algorithm, robotics, Google maps, etc.

CO 2. Apply knowledge of discrete mathematics in Computer Networking in many Government and Private agency.

CO 3. Use the Knowledge of Advanced Linear Algebra in the area of Geometry and other scientific subjects.

CO 4 To think and apply the basic knowledge of Fluid Mechanics, further research in this area and other applied Mathematics.

CO 5. The knowledge of Probability Theory use in day to day life problems and statical investigations.

CO 6. Solve problems related to the course in various competitive exams like IAS, PCS, faculty exams of higher education etc.

B030905T Elective-II

Any one of the following

Course Objective: The objective of these papers are to study Advanced Ordinary Differential Equation, Fuzzy Sets and Its Applications, Nonlinear Analysis and Special Function. Also to prepare the students for further research in these area.

Advanced Ordinary Differential Equations :

Unit I

Initial and Boundary Value Problems, Picards method of successive approximations, Lipschitz conditions, Sufficient conditions for being Lipschitzian in terms of partial derivatives, Examples of Lipschitzian and Non-Lipschitzian functions, Existence and Uniqueness theorem for first order initial value problem, Differential equations of first order not solvable for the derivative.

Unit II

Uniqueness of solutions with a given slope, Singular solutions, p- and c-discriminant equations of a differential equation and its family of solutions respectively, Envelopes of one parameter family of curves, singular solutions as envelopes of families of solution curves, Sufficient conditions for existence and non-existence of singular solutions, examples. Systems of I order equations arising out of equations of higher order, Norm of Euclidean spaces convenient for analysis of systems of equations, Lipschitz condition for functions from \mathbb{R}^{n+1} to \mathbb{R}^n , Local existence and uniqueness theorems for systems of I order equations, Gronwalls inequality, Global existence and uniqueness theorems for existence of unique solutions over whole of the given interval and over whole of \mathbb{R} , Existence theory for equations of higher order, Conditions for transformability of a system of I order equations into an equation of higher order.

Unit III

Linear independence and Wronskians, General solutions covering all solutions for homogeneous and non-homogeneous linear systems, Abels formula, Method of variation of parameters for particular solutions, Linear systems with constant coefficients, Matrix methods, Different cases involving diagonalizable and non-diagonalizable coefficient matrices, Real solutions of systems with complex eigenvalues.

Unit IV

Ordinary and singular points, Power series solutions, Frobenius generalized power series method, Indicial equation, different cases involving roots of the indicial equation, Regular and logarithmic solutions near regular singular points Legendres equation, Solution by power series method, polynomial solution, Legendre polynomial, Rodrigues formula, Generating function, Recurrence relations, Bessels equation, Bessel functions of I and II kind, Recurrence relations, Bessel functions of half-integral orders, Zeros of Bessel functions, Orthogonality relations, Generating function.

Books Recommended:

1. B. Rai, D. P. Choudhury and H. I. Freedman, A Course in Ordinary Differential Equations, Narosa Publishing House, New Delhi, 2002.
2. L. Collatz, The Numerical Treatment of differential equations, Springer-Verlag, 1960.
3. E. A. Coddington, An Introduction to Ordinary Differential Equations, Prentice Hall of India, New Delhi, 1968.
4. D.V. Widder, Advanced Calculus, Prentice Hall, 1961.

Fuzzy Sets and Its Applications :

Unit-I

Basic Concepts of Fuzzy Sets: Motivation, Fuzzy sets and their representations, Membership functions and their designing, Types of Fuzzy sets, Operations on fuzzy sets, Convex fuzzy sets. Alpha-level cuts, Zadehs extension principle, Geometric interpretation of fuzzy sets.

Unit-II

Fuzzy Relations: Fuzzy relations, Projections and cylindrical extensions, Fuzzy equivalence relations, Fuzzy compatibility relations, Fuzzy ordering relations, Composition of fuzzy relations. Fuzzy Arithmetic: Fuzzy numbers. Arithmetic operations on fuzzy numbers.

Unit-III

Fuzzy Logic: Fuzzy propositions, Fuzzy quantifiers, Linguistic variables, Fuzzy inference, Possibility Theory: Fuzzy measures, Possibility theory, Fuzzy sets and possibility theory, Possibility theory versus probability theory.

Unit-IV

Probability of a fuzzy event. Bayes theorem for fuzzy events. Probabilistic interpretation of fuzzy sets. Fuzzy mapping rules and fuzzy implication rules. Fuzzy rule-based models for function approximation. Types of fuzzy rule-based models (the Mamdani, TSK, and standard additive models). Fuzzy Implications and Approximate Reasoning: Decision making in Fuzzy environment: Fuzzy Decisions, Fuzzy Linear programming, Fuzzy Multi criteria analysis, Multi-objective decision making.

Recommended Books:

1. J. Yen and R. Langari: Fuzzy Logic: Intelligence, Control, and Information, Pearson Education, 2003.
2. G. J. Klir and B. Yuan: Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice-Hall of India, 1997.
3. H.J. Zimmermann, Fuzzy Set theory and its Applications, Kluwer Academic Publ, 2001.

Nonlinear Analysis:

Unit I

Compactness in Metric spaces, Measure of Noncompactness, Normed spaces, Banach spaces, Hilbert spaces, Uniformly convex, strictly convex and reflexive Banach spaces, Lipschitzian and contraction mapping, Banachs contraction principle, Application to Volterra and Fredholm integral equations.

Unit II

Nonexpansive, asymptotically nonexpansive, accretive and quasinonexpansive mappings, Fixed point theorems for nonexpansive mappings, Nonexpansive operators in Banach spaces satisfying Opials conditions, The demiclosedness principle.

Unit III

Schauders fixed point theorem. Condensing maps. Fixed points for condensing maps, The modulus of convexity and normal structure, radial retraction, Sadovskii's fixed point theorem, Set-valued mappings.

Unit IV

Fixed point iteration procedures, The Mann Iteration, Lipschitzian and Pseudocontractive operators in Hilbert spaces, Strongly pseudocontractive operators in Banach spaces, The Ishikawa iteration,

Stability of fixed point iteration procedures. Iterative solution of Nonlinear operator equations in arbitrary and smooth Banach spaces, Nonlinear m-accretive operator, Equations in reflexive Banach spaces.

Books Recommended:

1. V. Berinde, Iterative Approximation of Fixed Points, Lecture Notes in Mathematics, No. 1912, Springer, 2007.
2. M. A. Khamsi and W. A. Kirk, An Introduction to Metric Spaces and Fixed Point Theory, John Wiley & Sons, New York, 2001.
3. Sankatha P. Singh, B. Watson and P. Srivastava, Fixed Point Theory and Best Approximation: The KKM-map Principle, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1997.
4. V. I. Istratescu, Fixed Point Theory, An Introduction, D. Reidel Publishing Co., 1981.
5. K. Goebel and W. A. Kirk, Topic in Metric Fixed Point Theory, Cambridge University Press, 1990.

Special Function:

Unit I

The Hypergeometric function: An integral representation. Its differential equation and solutions. , $F(a,b,c;1)$ as a function of the parameters, evaluation of $F(a,b,c;1)$, contiguous function relations, the hypergeometric differential equation, logarithmic solutions of the hypergeometric equation.

Unit II

$F(a,b,c;z)$ as a function of its parameters, Elementary series manipulations, simple transformations, relation between functions of (z) and, $(1-z)$ quadratic transformations, theorem due to Kummer, additional properties. The Confluent Hypergeometric function: Basic properties of ${}_1F_1$, Kummers first formula. Kummers second formula.

Unit III

Generalized Hypergeometric Series: The function ${}_pF_q$, the exponential and binomial functions, differential equation, contiguous function relations, integral representation ${}_pF_q$, with unit argument, Saalshutz theorem, Whipples theorem, Dixons theorem, Contour integrals of Barnes type.

Unit IV

Bessel Functions: Definition, Differential equation, differential recurrence relations, pure recurrence relation, generating function, Bessels Integral, index half an odd integer, modified Bessel functions. Introduction to Legendre function.

Books Recommended:

1. Earl. D. Ranvillie, Special Functions , Macmillan, 1960.
- 2.L.C. Andrews ,Special Functions of Mathematics for Engineers, SPIE Press, 1992.
3. Gabor Szego, Orthogonal Polynomials, American mathematical society, 1939.
4. L.J. Slater,Generalized Hypergeometric Functions , Cambridge University Press; Reissue edition ,2008.

Course Outcomes: After the completion of these courses, the student will be able to

CO 1. Apply Advanced Ordinary differential equation to understand deeper issue in the area of applied mathematics.

CO 2. Understand applications of Fuzzy Sets and Its Applications .

CO 3. Think and use of Nonlinear Analysis in other branches.

CO 4. Understand the basic knowledge of special functions so that further use in Advanced level and research.

CO 5. Solve problems related to the course in various competitive exams like IAS, PCS, faculty exams of higher education etc.

31. Detailed Syllabus for Semester-X of P.G.(Mathematics):

B031001T Measure and Integration:

Course Objectives: A measure is a generalization of the concept of length, area and volume. The main objective of this paper is that to prepare the students for further research in measure & Integration theory and analysis.

Unit I

Cardinality of a set, Arithmetic of cardinal numbers, Schröder-Bernstein theorem, The Cantor ternary set and its properties, The Cantor-Lebesgue function. Semi-algebras, algebras, monotone class, σ -algebras, measure and outer measures, Carathéodory extension process of extending a measure on semi-algebra to generated σ -algebra, completion of a measure space.

Unit II

Borel sets, Lebesgue outer measure and Lebesgue measure on \mathbb{R} , translation invariance of Lebesgue measure, existence of a non-measurable set, characterizations of Lebesgue measurable sets.

Unit III

Measurable functions, Characterization of measurable functions, Linearity and products of measurable functions, Borel and Lebesgue measurable functions, Characteristic functions, simple functions and their integrals, Lebesgue integral on \mathbb{R} and its properties, Characterizations of Riemann and Lebesgue integrability.

Unit IV

Littlewoods three principles (statement only), Bounded convergence theorem, Lebesgue monotone convergence theorem, Fatous lemma, Lebesgue dominated convergence theorem.

Books Recommended:

1. K. Rana, An Introduction to Measure and Integration, Second Edition, Narosa Publishing House, New Delhi, 2005.
2. P. R. Halmos, Measure Theory, Grand Text Mathematics, 14, Springer, 1994.
3. E. Hewit and K. Stromberg, Real and Abstract Analysis, Springer, 1975.
4. K. R. Parthasarathy, Introduction to Probability and Measure, TRIM 33, Hindustan Book Agency, New Delhi, 2005.
5. H. L. Royden and P. M. Fitzpatrick, Real Analysis, Fourth edition, Prentice Hall of India, 2010.

Course Outcomes: After the completion of the course, the student will be able to

CO 1. Apply measure theory in Statistics to understand deeper issue in probability theory.

CO 2. Understand applications of measure theory in economics .

CO 3. Solve problems related to the course in various competitive exams like IAS, PCS, faculty exams of higher education etc.

B031002T Elective-I

Any one of the following

Course Objective: The objective of these papers are to study Tychonoff product topology in terms of standard sub-base and its characterizations Characterization, Homotopy of paths, classification of covering spaces with the fundamental group. Archimedean and non-Archimedean absolute values, approximation theorem in the Algebraic Number Theory, Simple situations requiring mathematical modelling, techniques of mathematical modelling. Also to prepare the students for further research in Topology, Algebraic Number Theory, Mathematical Modelling and Advanced Partial Differential Equations.

Advanced Topology

Unit I

Tychonoff product topology in terms of standard sub-base and its characterizations. Projection maps. Separation axioms and product spaces. Connectedness and product spaces. Compactness and product spaces (Tychonoff's theorem). Countability and product spaces. Embedding and metrization. Embedding lemma and Tychonoff embedding. The Urysohn metrization theorem.

Unit II

The Stone-Cech compactification. Paracompact spaces, their properties and characterizations. Metrizable spaces and Metrization theorems. Uniform spaces, Weak uniformity, Uniformizability. Completion of uniform spaces. Function spaces. Point-wise and uniform convergence. The compact open Topology. The Stone-Weierstrass theorem.

Unit III

Homotopy and homotopy type, cell complexes, homotopy extension property. The fundamental group: homotopy of paths, homotopy-lifting property for paths through covering spaces, functoriality, Van Kampens theorem, covering spaces. classification of covering spaces with the fundamental group, deck transformations and group actions, $K(G, 1)$ s.

Unit IV

Singular homology, cellular homology MayerVietoris Sequences, homology with coefficients, relation with the fundamental group, relative homology, the long exact sequence of relative homology, applications to computing the homology of surfaces, projective spaces, axioms for homology, Euler characteristic.

Books Recommended :

1. James R. Munkers, Topology, A First Course, Prentice-Hall of India Pvt. Ltd. New Delhi. 200.

2. George F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill Book Company, 1963.
3. K.D. Joshi, Introduction to General Topology, Wiley Eastern Ltd. 1983.
4. J. Hocking and G. Young, Topology, Addison-Wesley, Reading, 1981.
5. W.J. Pervin, Foundations of General Topology, Academic Press Inc. New York, 1964.

Further Reading:

1. Algebraic Topology by Allen Hatcher.

Algebraic Number Theory

Unit I

Archimedean and non-Archimedean absolute values, approximation theorem, absolute values on \mathbb{Q} and $F[x]$, completion of a field, Ostrowski's theorem.

Unit II

p -adic numbers and p -adic integers, arithmetic in \mathbb{Q}_p , Hensel's lemma, sequence and series in \mathbb{Q}_p , exponential and logarithmic series in \mathbb{Q}_p .

Unit III

Number fields, the ring of algebraic integers, calculation for quadratic, cubic and cyclotomic case, norms and traces, discriminants.

Unit IV

Dedekind domains, unique factorization of ideals, splitting of primes in extensions, decomposition and inertia group, unramified and ramified extensions, the Frobenius automorphism. The ideal class group, lattices in \mathbb{R}^n , finiteness of the class number, the Dirichlet unit theorem.

Books Recommended:

1. J. S. Milne, Algebraic Number Theory, 2011.
2. N. Jacobson, Basic Algebra, Vol. 2, Hindustan Publishing Corporation, New Delhi, 1994.
3. D. A. Marcus, Number Fields, Springer-Verlag, 1977.
4. J. Esmonde and M. Ram Murty, Problems in Algebraic Number Theory, Graduate Text in Mathematics, 190, Springer-Verlag, 1999.
5. A. M. Roberts, A Course in p -adic Analysis, Graduate Text in Mathematics, 198, Springer-Verlag, 2000.

Mathematical Modelling:

Unit I

Simple situations requiring mathematical modeling, techniques of mathematical modeling, Classifications, Characteristics and limitations of mathematical models, Some simple illustrations.

Unit II

Mathematical modeling through differential equations, linear growth and decay models, Non linear growth and decay models, Mathematical modeling in dynamics through ordinary differential equations of first order.

Unit III

Mathematical models through difference equations, some simple models, Basic theory of linear difference equations with constant coefficients, Mathematical modeling through difference equations in

economic and finance, Mathematical modeling through difference equations in population dynamic.

Unit IV

Mathematical modeling through linear programming, Linear programming models in Transportation and assignment.

Recommended Books:

1. J. N. Kapur, Mathematical Modeling, Wiley Eastern.
2. D. N. Burghes, Mathematical Modeling in the Social Management and Life Science, Ellie Herwood and John Wiley.
3. F. Charlton, Ordinary Differential and Difference Equations, Van Nostrand.

Advanced Partial Differential Equation

Unit I

Laplace's equation- Fundamental solution. Mean Value Formula Properties of Harmonic Functions. Green's Function. Energy Methods. Heat Equation- Fundamental Solution. Mean Value Formula. Properties of Solutions Energy Methods.

Unit II

Wave Equation- Solution by Spherical Mean, Non-homogeneous Equations. Energy Methods. Non-linear First Order PDE-Complete Integrals, Envelopes, Characteristics. Hamilton Jacobi Equations (Calculus of Variations. Hamilton's ODE. Legendre Transform. Hopf-Lax Formula. Weak Solutions. Uniqueness).

Unit III

Conservation Laws (Shocks, Entropy Condition, Lax-Oleinik Formula, Weak Solutions, Uniqueness, Riemann's Problem. Long Time Behaviour). Representation of Solutions- Separation of Variables. Similarity Solutions (Plane and Travelling Waves).

Unit IV

Solitons, Similarity under Scaling), Fourier and Laplace Transform. Hopf-Cole Transform, Hodograph and Legendre Transforms, Potential Functions, Asymptotic (Singular Perturbations, Laplace's Method. Geometric Optics. Stationary Phase, Homogenization). Power Series (Non-characteristic surface. Real Analytic Functions, Cauchy-Kovalevskaya Theorem).

Books Recommended :

1. I.M. Gelfand and S.V. Fomin, Calculus of Variations, Prentice Hall.
2. A.M. Arthurs, Complementary Variations Principle, Clarendon Press, Oxford, 1970.
3. A.s. Gupta, Calculus of Variations with Applications, Prentice Hall of India, 1997.
4. L.C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Volume 19, AMS, 1998.
5. I.N. Sendon, Elements of Partial Differential Equations, McGraw Hill Book Co., 1988.
6. P. Prasad and R. Ravindran, Partial Differential Equations.

Course Outcome: After the completion of the course, the student shall be able to

CO 1. Understand the basic of the courses, Advanced Topology, Algebraic Number Theory, Mathematical Modelling & Advanced Partial differential equation and think & develop new ideas in these

courses.

CO 2. Get involved in wide area of research in analysis and differential geometry with the help of Advanced Topology.

CO 3. Using the Knowledge of Algebraic Number theory in the research of number theory and coding theory.

CO 4. Make predictions of the behaviour of a given system based on the analysis of its mathematical model.

CO 5. Apply the knowledge of Advanced Partial differential equation to solve the physical problems.

B031003T Elective-II

Course Objectives: The objective of these papers are to study Advanced Special Function, Wavelet Analysis, Mathematical Method and Module Theory. The aim of Advanced Special function is to extend the knowledge special function and utilised it in research, and in case of wavelet analysis our purpose to study of Fourier and inverse Fourier transforms convolution and delta function, Fourier transform of square integrable functions, wavelet transform, time frequency Analysis, Gabor transform, Dyadic wavelets and inversion, Wavelet series, Scaling functions and wavelets, Multi resolution analysis, orthogonal wavelets and wavelet Backet, Examples of orthogonal wavelets, orthogonal wavelet packets, orthogonal decomposition of wavelet series. The main objective of this paper is that to prepare the students for further research in wavelet analysis, and the extend the knowledge of Mathematical Method and Module Theory in Advanced label, so that it use in research.

Any one of the following

Advanced Special Function:

Unit I

Weierstrass Elliptic Functions: Periodic functions, Lower bound of the periods of an analytic function Jacobi's first and second questions, definition of an elliptic function, The irreducible poles and zeros of an elliptic function.

Unit II

Weierstrass's sigma functions and zeta functions , Pseudo-periodicity of zeta and sigma functions. The algebraic relation connecting two elliptic functions, differential equation satisfies by $\wp(z)$, The constants e_1, e_2, e_3 solution of a differential equations, addition theorem for $\wp(z)$, a formula for $\wp(z+w)$ in terms of $\wp(z)$.

Unit III

The expression of an elliptic function in terms of sigma function, The function of $\sigma(z)$. The expression of an elliptic functions in terms of zeta functions, The expression of an elliptic function in term of $\sigma(z)$, Jocabi's Elliptic Function: The construction of elliptic functions with two simple poles each cell, General description of the function $S(z)$, $C(z)$ and $D(z)$.

Unit IV

The complementary modulus, Jacobi's elliptic function, The general properties of the $s_n(u)$, $c_n(u)$ and $d_n(u)$
 The addition theorem Jacobi's elliptic function of given modulus, Evaluation of an elliptic integral,
 Expression of K in term of a hypergeometric function, The expression of K in terms of k .

Recommended Books:

1. Copson, E. T. ; An Introduction to the Theory of Function of a Complex Variable, Oxford University Press, 1935.
2. Whittaker and Watson ; Modern Analysis Chapter XI (11.1-11.31).

Wavelet Analysis:

UNIT I

Multi-resolution analysis and MRA wavelets, Low pass filter, Characterizations in multiresolution analysis, compactly supported wavelets, band-limited wavelets.

UNIT II

Franklin wavelets on \mathbb{R} , Dimension function, Characterization of MRA wavelets (Sketch of the proof), Minimally Supported Wavelets, Wavelet Sets, Characterization of two-interval wavelet sets, Shannon wavelet, Journes wavelet, Decomposition and reconstruction algorithms of Wavelets.

Unit III

Wavelet Transforms and Time Frequency Analysis: The Gabor Transform. Short-time Fourier transforms and the uncertainty principle. The integral wavelet transforms Dyadic wavelets and inversions. Frames. Wavelet Series. Scaling Functions and Wavelets: Multi resolution analysis, scaling functions with finite two scale relations. Direct sum decomposition of $L^2(\mathbb{R})$.

Unit IV

Linear phase filtering, Compactly supported wavelets, Wavelets and their duals, Orthogonal Wavelets and Wavelet packets, Example of orthogonal Wavelets. Identification of orthogonal two-scale symbols, Construction of Compactly supported orthogonal wavelets, Orthogonal wavelet packets, orthogonal decomposition of wavelet series.

Books Recommended:

1. Eugenio Hernandez and Guido Weiss, A First Course on Wavelets, CRC Press, 1996.
2. Ingrid Daubechies, Ten Lectures on Wavelets, CBS-NFS Regional Conferences in Applied Mathematics, 61, SIAM, 1992.
3. Michael W. Frazier, An Introduction to Wavelets through Linear Algebra, Springer-Verlag, 1999.
4. C. K. Chui, An Introduction to Wavelets, Academic Press, 1992.

Mathematical Method:

Unit I

Periodic functions, Trigonometric series, Fourier series, Euler formulas, Functions having arbitrary periods, Even and Odd functions, Half-range expansions, Determination of Fourier coefficients without integration, Approximation by trigonometric polynomials, Square error. Orthogonal and Orthonormal sets of functions, Generalized Fourier series, Sturm-Liouville problems, Examples of Boundary-value problems which are not Sturm-Liouville problems, Greens functions.

Unit II

Laplace Transform, Properties of Laplace transform, Laplace Transform of a periodic function, error function and Dirac Delta function, Inverse transform Shifting and linearity property of Laplace transform, Laplace Transform of the derivatives and of the Integrals of a function, Derivatives and Integrals of Transforms, Convolution theorem (Faltung theorem).

Unit III

Fourier Transform, Properties of Fourier transform (linearity property, change of scale, shifting property, modulation property), Fourier Integrals, Fourier Cosine and Sine Integrals, Inverse Fourier Transform, Fourier Cosine and Sine Transform, Complex form of the Fourier Transform, Convolution theorem.

Unit IV

Finite Fourier transform, Finite sine transform, Finite Cosine transform, Solution of ordinary differential equations using Laplace transform method, Solution of wave equation using the method of Fourier transform: D'Alembert's solution of the wave equation.

Books Recommended:

1. E. Kreyszig, Advanced Engineering Mathematics, Wiley India Pvt. Ltd., 8th Edition, 2001.
2. J. H. Davis, Methods of Applied Mathematics with a MATLAB Overview, Birkhuser, Inc., Boston, MA, 2004.

Module Theory:

Unit I

Modules over a ring, Endomorphism ring of an abelian group, R-Module structure on an abelian group M as a ring homomorphism from R to $\text{End}_{\mathbb{Z}}(M)$, submodules, Direct summands, Annihilators, Faithful modules, Homomorphism, Factor modules, Statements of Correspondence theorem and Isomorphism theorems, $\text{Hom}_R(M, N)$ as an abelian group and $\text{Hom}_R(M, M)$ as a ring, Exact sequences, Five lemma, External and internal direct sums and their universal property.

Unit II

Free modules, Homomorphism extension property, equivalent characterization as a direct sum of copies of the underlying ring, existence of a basis of a vector space, Split exact sequences and their characterizations, Left exactness of Hom sequences and counter-examples for non-right exactness, Projective modules, Injective modules, Baer's characterization, Divisible groups, Examples of injective modules.

Unit III

Factorization theory in commutative domains, Prime and irreducible elements, G.C.D., Euclidean domains, Maximal and prime ideals, Principal ideal domains, Divisor chain condition, Unique factorization domains, Examples and counterexamples, Chinese remainder theorem for rings and PIDs, Polynomial rings over domains, Unique factorization in polynomial rings over UFDs.

Unit IV

Submodules of finitely generated free modules over a PID, Torsion submodule, Torsion and torsion-free modules, Direct decomposition into $T(M)$ and a free module, \mathfrak{p} -primary components, Decomposition of \mathfrak{p} -primary finitely generated torsion modules, Elementary divisors and their uniqueness, Decomposition into invariant factors and uniqueness, Direct sum decomposition of finite abelian groups into cyclic groups and their enumeration. Reduction of matrices over polynomial rings over a field, Similarity of matrices and $F[x]$ -module structure, Rational canonical form of matrices, Elementary Jordan matrices, Reduction to Jordan canonical form, Diagonalizable and nilpotent parts

of a linear operator, Jordan-Chevalley Theorem

Books Recommended:

1. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley, N.Y., 2003.
2. F. W. Anderson and K. R. Fuller, Rings and Categories of Modules, Springer, N.Y., 1974.
3. I. A. Adamson, An Introduction to Field Theory. Oliver and Boyd, Edinburgh, 1964.
4. N. S. Gopalakrishnan, University Algebra, Wiley Eastern Ltd., New Delhi, 1986.

Further Reading:

1. T. W. Hungerford, Algebra, Springer (India) Pvt. Ltd., New Delhi, 2004.
2. P. Ribenboim, Rings and Modules, Wiley Interscience, N.Y., 1969.
3. J. Lambek, Lectures on Rings and Modules, Blaisedell, Waltham, 1966.
4. Ramji Lal, Algebra, Vols. II, Shail Publications, Allahabad, 2002.

Course Outcomes: After the completion of the course, the student shall be able to

CO 1. Understand the basics of these courses.

CO 2. Think and develop new ideas in these fields.

CO 3. Understand the use of this courses in science and technology and other fields of Mathematical Analysis.

B031004 Elective-III

Any one of the following

Course Objective: The objective of papers Differential Geometry of Manifolds, Dynamical System, General relativity & Cosmology and operator theory to provide the wide area of study in Mathematics and use these knowledge to further utilized in research.

Differential Geometry of Manifolds:

Unit I

Topological manifolds, compatible charts, smooth manifolds, examples, smooth maps and diffeomorphisms, definition of a Lie group, examples. Tangent and cotangent spaces to a manifold, Derivative of a smooth map, Immersions and submersions, Submanifolds, vector fields, algebra of vector fields, -related vector fields, left and right invariant vector fields on Lie groups.

Unit II

Integral curves of smooth vector fields, complete vector fields, flow of a vector field, Distributions, n-dimensional real vector space, Contravariant vectors, Dual vector space, Covariant vectors.

Unit III

Tensor product of vector spaces, Second order tensors, Tensors of type (r, s) , Symmetry and skew symmetry of tensors, Fundamental algebraic operations, Quotient law of tensors, Tensor fields on

manifolds, Differential forms, Exterior product, Exterior differentiation, Pull-back differential forms.

Unit IV

Affine connections (covariant differentiation) on a smooth manifold, torsion and curvature tensors of an affine connection, Identities satisfied by curvature tensor.

Books Recommended:

1. S. Kobayashi and K. Nomizu, Foundations of Differential Geometry, Vol. 1, Interscience Publishers, 1963.
2. T. J. Willmore, Riemannian Geometry, Oxford Science Publication, 1993.
3. S. Kumaresan, A Course in Differential Geometry and Lie Groups, Hindustan Book Agency, New Delhi, 2002.
4. M. Spivak, A Comprehensive Introduction to Differential Geometry, Vols. 1-5, Publish or Perish, Inc., Houston, 1999.
5. W. M. Boothby, An Introduction to Differentiable Manifolds and Riemannian Geometry, Academic Press, revised, 2003.

Dynamical System:

UNIT I:

Dynamical systems, Iterates of a function, trajectories and orbits, recursion equations, phase portraits, the logistic function. Review of metric spaces, topology of \mathbb{R} and analysis of real functions, fixed (or equilibrium) points, periodic points, asymptotic points, stable sets, graphical analysis.

UNIT II:

Sarkovskii's ordering, Sarkovskii's theorem, sufficient conditions for a function on a closed interval to have a unique fixed point, dynamical information from a differentiable function, hyperbolic and nonhyperbolic periodic points, attracting periodic points.

UNIT III:

Sigma-algebras and subalgebras, measure algebras, atoms of a measure algebra and the Caratheodory's theorem, symbol space, shift maps, topologically transitive functions, sensitive dependence, chaotic functions, topological conjugates.

UNIT IV:

Measure preserving transformations, definitions and examples, construction of a new measure preserving transformation from given ones, homomorphisms, isomorphisms, recurrence property of a measure preserving transformation, Poincaré's recurrence theorem, introduction to ergodicity.

Books Recommended:

1. R. A. Holmgren, A First Course in Discrete Dynamical Systems, Springer-Verlag, 1994.
2. H. L. Royden and P. M. Fitzpatrick, Real Analysis (Fourth Edition), Prentice Hall, 2010.
3. P. Walters, An Introduction to Ergodic Theory, Springer, 1982.

Further Reading:

1. R. L. Devaney, An Introduction to Chaotic Dynamical Systems, Second Edition, Addison- Wes-

ley, 1989.

2. K. R. Parthasarthy, Introduction to Probability and Measure, Hindustan Book Agency, 2005.

General relativity and Cosmology:

Unit I

General Relativity- Transformation of coordinates. Tensors. Algebra of Tensors. Symmetric and skew symmetric Tensors. Contraction of tensors and quotient law. Riemannian metric, Parallel transport, Christoffel Symbols. Covariant derivatives. Intrinsic derivatives and geodesics, Riemann Christoffel curvature tensor and its symmetry properties. Bianchi identities and Einstein tensor.

Unit II

Review of the special theory of relativity and the Newtonian Theory of gravitation. Principle of equivalence and general covariance, geodesic principle. Newtonian approximation. Schwarzschild external solution and its isotropic form. Planetary orbits and analogues of Kepler's laws in general relativity. Advance or perihelion of a planet. Bending of light rays in gravitational field. Gravitational redshift of spectral lines. Reader echo delay.

Unit III

Energy- momentum tensor of a perfect fluid. Schwarzschild internal solution. Boundary conditions. Energy momentum tensor of an electromagnetic field. Einstein-Maxwell equations. Reissner-Nordstrom solution. Cosmology- Mach's principle. Einstein modified field equations with cosmological term. Static Cosmological models of Einstein and De-Sitter, their derivation, properties and comparison with the actual universe.

Unit IV

Hubble's law. Cosmological principle's Weyl's postulate. Derivation of Robertson-Walker metric. Hubble and deceleration parameters. Redshift. Redshift versus distance relation. Angular size versus redshift relation and source counts in Robertson- Walker space-time, Friedmann models.

Books Recommended

1. C.E. Weatherburn An Introduction to Riemannian Geometry and the tensor Calculus, Cambridge University Press, 1950.
2. J.V. Narlikar, General Relativity and Cosmology, The Macmillan Company of India Ltd. 1978.
3. B.F. Schutz, A first course in general relativity, Cambridge University Press, 1990.
4. A.S. Eddington, The Mathematical Theory of Relativity, Cambridge University Press, 1965.
5. S. Weinberg Gravitation and Cosmology : Principle and applications of the general theory of relativity, John Wiley & Sons, Inc. 1972.
6. J.V. Narlikar, Introduction to Cosmology, Cambridge University Press, 1993.

Operator Theory:

Unit I

Spectral Theory of Linear Operators in Normed Spaces, Spectral Theory in Finite Dimensional Normed Spaces.

Unit II

Basic Concepts, Spectral properties of Bounded Linear Operators, Further Properties of Resolvent and Spectrum.

Unit III

Use of Complex Analysis in Spectral Theory. Banach Algebras, Further properties of Banach Algebra.

Unit IV

Gelfand- Naimark theorem. Non-commutative C^* -algebras and Gelfand-Naimark-Segal construction.

Recommended Books:

- 1 E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley and Sons.
- 2 John B. Conway, A Course in Operator Theory, Springer.
- 3 G.Bachman, L. Narici, Functional analysis, Academic Press, N.Y.
- 4 G. F. Simmons, An Introduction to Topology and Modern Analysis, Tata McGrawHill.

Course Outcomes: After the completion of the course, the student shall be able to

CO 1. Understand the basics of the courses Differential Geometry of Manifolds, Dynamical System, General relativity and Cosmology and operator theory .

CO 2. Think and develop new ideas in these fields Differential Geometry of Manifolds, Dynamical System, General relativity & Cosmology and operator theory.

CO 3. Understand the use of this courses in Geometry, science and technology and other fields of Mathematical Analysis.

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