Question Bank

Subject: Operations research *(EOE*-073*)*

Unit-I: Introduction and LPP

Q1- Discuss the statement that OR is Science and an Art both?

Q2- State the different types of models used in OR. Explain briefly the general methods for Solving these OR models.

Q3- Give the mathematical and economic structure of linear programming problems?

Q4 - A firm manufactures two products A and B with a profit ₹4 and ₹5 respectively. The products are made by the machines X and Y. The available machine time in minutes for the machines X and Y are 2000 and 1500 respectively. The time taken to make the product A by machines X and Y are 5 and 4 minutes and for the product B are 3 and 4 minutes respectively. Formulate the LP model for maximization of the profit.

Ans- Max. $z= 4x_1+5x_2$

Subject to

$$5x_1+ 3x_2 \leq 2000$$
$$4x_1+ 4x_2 \leq 1500$$
$$X_1, x_2 \geq 0$$

Q5- Use the graphical method to solve the following LP problem:

Max. $Z = X+ Y$

Subject to the constraints,

$X + Y \leq 1,$

$-3X + Y \geq 3$, and $X, Y \geq 0$. **(Ans-the solution is infeasible)**

Q6- Use the graphical method to solve the following LP problem:

Max. $Z = X+ 3Y$

Subject to the constraints,

$X + Y \leq 2,$

$-X + Y \leq 4,$

X is unrestricted, and $Y \geq 0$. **(Ans-at (1,3) & 8)**

Q7- Use the simplex method to solve the following LP problem:

Min. $Z=X -3 Y +2 Z$

Subject to the constraints,

$3X - Y+3Z \leq 7, -2 X +4Y \leq 12, -4X +3 Y +8 Z \leq 10$

And $X, Y, Z \geq 0$ **(Ans-at (4,5,0) & -11)**

Q8- Use two-phase method to solve the following LP problem:

Maximize $Z= 3X - Y$

Subject to the constraints:

$2X + Y \geq 2, X + 3 Y \leq 2, Y \leq 4$ and $X, Y \geq 0$

(Ans- (2,0) & 6)

Q9- Use two-phase method to solve the following LP problem:

Min. $Z= 15/2X - 3Y$

Subject to the constraints:

$3X – Y-Z \geq 3, X – Y+Z \geq 2$, and $X, Y, Z \geq 0$

(Ans- (5/4,0,3/4) & 75/8)

Q10- Use Big-M method to solve the following LP problem:

Max. $Z = -2X - Y$

Subject to the constraints:

$3X + Y = 3,\ 4X + 3Y \geq 6,\ X + 2Y \leq 4$ and $X, Y \geq 0$

(Ans-(3/5,6/5) & Z= -12/5)

Q11- Use Big-M method to solve the following LP problem:

Max. $Z = 2X + Y + 3Z$

Subject to the constraints:

$X + Y + 2Z \leq 5,\ 2X + 3Y + 4Z = 12$, and $X, Y, Z \geq 0$

(Ans-(3,2,0) & Z=8)

Q12- Explain in brief the Primal and Dual problems.

Q13- Solve the following problem by dual simplex method.

Min. $z = 2x_1 + x_2$

Subject to $3x_1 + x_2 \geq 3,\ 4x_1 + 3x_2 \geq 6,\ x_1 + 2x_2 \geq 3$

and $x_1, x_2 \geq 0$.

Ans- $z = 12/5$ with $x_1 = 3/5$, $x_2 = 6/5$ and $x_3 = x_4 = x_5 = 0$.

Q14- solve by using duality theory:

Max. $z = 2x_1 + x_2$

Subject to the constraints: $x_1 + x_2 \leq 2$

$x_1 + 3x_2 \leq 3$ **Ans- $x_1 = 4$, $x_2 = 2$, Max. $z = 10$**

$x_1, x_2 \geq 0$

Q15- Solve the following LPP by using revised simplex method:

Max. $z = x_1 + 2x_2$

Subject to $x_1 + 2x_2 \leq 3$

$x_1 + 2x_2 \leq 1$

$x_1, x_2 \geq 0$ **Ans- $x_1 = 1$, $x_2 = 0$, $z = 1$**

Unit-II: Introduction about Transportation and Assignment problems.

Q1- A department head has four subordinates and four tasks have to be performed. Subordinates differ in efficiency and tasks differ in their intrinsic difficulty. Time each man would take to perform each task is given in the effectiveness matrix. How the tasks should be allocated to each person so as to minimize the total man hours?

Subordinates

	I	II	III	IV
A	8	26 d	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

Ans-

Optimal assignment:	**A→I**	**B→III**	**C→II**	**D→IV**
Man power:	**8**	**4**	**19**	**10**

Q2- Solve the following transportation problem for minimum cost.

To

					ai
From	15	10	17	18	2
	16	13	12	13	6
	12	17	20	11	7
bj	3	3	4	5	

Ans-174

Q3- Describe the degeneracy in transportations problems?

Q4- Solve the following transportation problem:

	To				Supply
From	6	1	9	8	70
	11	5	2	8	55
	10	12	4	7	90
Demand	85	35	50	45	

Ans-1265

Q5- A marketing manager has 5 salesman and 5 sales districts. Considering the capabilities of the salesman and the nature of districts, the marketing manager estimates that sales per month (In hundred rupees) for each salesman in each district would be as follows:

	A	B	C	D	E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

Find the assignment of salesman to districts that will result in maximum sale. (**Ans-191)**

Q6- Use Vogel's Approximation method. Find the initial basic feasible solution for the following transportation problem.

	To			
	A	B	C	Availability
From a	2	7	4	5
b	3	3	1	8
c	5	4	7	7
d	1	6	2	14
Requirement	7	9	18	34

Ans- 80(10+8+20+2+12+28)

Q7- Using Least Cost Rule, find the initial basic feasible solution of the following transportation problem.

To

	D1	D2	D3	D4	Availability
From A	2	2	2	1	7
B	10	8	5	4	9
C	7	6	6	8	18
Requirement	5	8	7	14	

Ans-158(7+28+10+30+48+35)

Q8- Determine an initial basic feasible solution to the following transportation problem by using North-West corner rule.

To

	D1	D2	D3	D4	D5	Supply
From A	2	11	10	3	7	4
B	1	4	7	2	1	8
C	3	9	4	8	12	9
Demand	3	3	4	5	6	21

Ans- 153(6+11+8+28+4+24+72)

Q9- Explain Vogel's Approximation method of solving a transportation method.

Q10- Explain North-West corner rule of solving a transportation method.

Q11- Explain the lowest cost entry method for obtaining an initial basic solution of a Transportation problem.

Q12- Find the optimal solution of the following transportation problem using Vogel's Approximation Method and find the initial basic feasible solution.

	D1	D2	D3	D4	Availability
From A	19	30	50	10	7
B	70	30	40	60	9
	40	8	70	20	18
Requirement	5	8	7	14	

Ans-743, 779

Unit-III: Introduction about Network techniques and Project Management PERT, CPM.

Q1- Write note on shortest path model.

Q2- Explain about Max-Flow problem and Min-cost problem.

Q3- Discuss various steps involved in the applications of PERT and CPM.

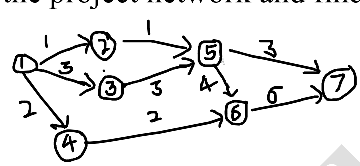
Q4- Explain about the network techniques?

Q5- Describe about the minimum-spanning tree problem.

Q6- A certain project is composed of mini activities whose time estimates are given below:

Activity	1-2	1-3	1-4	2-5	3-5	4-6	5-6	6-7	5-7
Duration	1	3	2	1	3	2	4	6	3

Draw the project network and find out the critical path.

Ans- 

Critical path 1-3-5-6-7.

Q7- State the circumstances where CPM is a better technique of project management than PERT.

Q8- How does PERT technique help a business manager in decision-making?

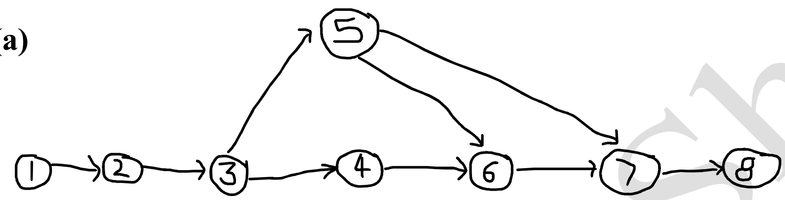
Q9- A research and development is developing a new power supply for a console television set. It has broken the project into the following jobs with their duration times in days:

Job	Immediate Predecessors	Duration time (in days)

A	-	5
B	A	7
C	B	2
D	B	3
E	C	1
F	D	2
G	C	1
H	E, F	3
I	G, H	10

(a) Draw a critical path scheduling diagram, identifying jobs letters and associated times with each. Indicate the critical path.
(b) What is the minimum time for the completion of the project?

Ans- (a)



Critical path: 1-2-3-4-6-7-8, i.e. A-B-D-F-H-I
(c) Minimum time for the completion of the project=30 days

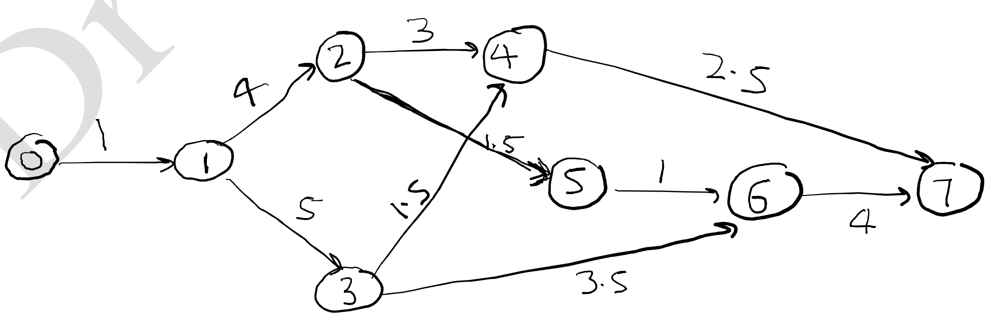
Q10- Compare and contrast CPM and PERT. Under what conditions would you recommend Scheduling by PERT? Justify your answer with reasons.

Q11- What are the major limitations of the PERT model? Discuss

Q12- Explain the following in the context of project management:
(i) Activity Variance
(ii) Project Variance

Diagram.net

Q13-



Determine the total float, free float, independent float and identify the critical path.

Ans- Activity:	**0-1**	**1-2**	**1-3**	**2-4**	**2-5**	**3-4**	**3-6**	**4-7**	**5-6**	**6-7**
T. float:	**0**	**2**	**0**	**3**	**2**	**3.5**	**0**	**3**	**2**	**0**

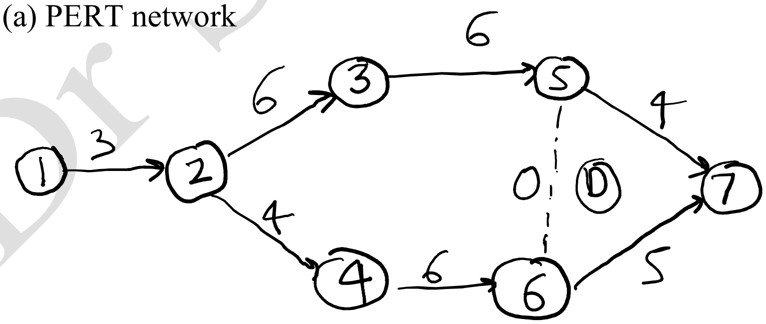
F. float:	**0**	**0**	**0**	**0**	**0**	**0.5**	**0**	**3**	**2**	**0**
I. float:	**0**	**0**	**0**	**-2**	**-2**	**0.5**	**0**	**0**	**0**	**0**

Q-14: A project schedule has the following characteristics.

Activity	Expected duration (Weeks)		
	Optimistic	Most likely	Pessimistic
1-2	3	3	3
2-3	3	6	9
2-4	2	4	6
3-5	4	6	8
4-6	4	6	8
5-6	0	0	0
5-7	3	4	5
6-7	2	5	8

(a) Draw the project network and trace all the possible paths from it.
(b) Determine the critical path and the expected project time.
(c) What is the probability that the project will be completed in 20 weeks?
(d) What is the probability that the project will be completed in 23 weeks?

Ans- (a) PERT network



All possible paths: 1-2-3-5-7, 1-2-3-5-6-7, 1-2-4-6-7
(b) critical path: 1-2-3-5-6-7. Expected project time 20 weeks.
(c) 0.5 i.e. 50%
(d) 0.9726 i.e. 97.26%

Unit-IV: Theory of Games.

Q1- Two players A and B, without showing each other, put a coin of Rs. 1, on a table, with head or tail up. If the coin show the same side(both head or tail), the player a takes both the coin ; otherwise B gets them. Construct the matrix of the game and solve it. Is it a fair game ?

Ans :
$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, A (1/2, 1/2), B (1/2,1/2), v=0. Yes

Q2- Discuss the Erlang distribution and poisson distribution.

Q3- Explain the principle of dominance in game theory and solve the following game:

Player B

Player A	B_1	B_2	B_3	B_4
A_1	8	10	9	14
A_2	10	11	8	12
A_3	13	12	14	13

Ans-A-III, B-II, v=12

Q4- Explain the Following terms:
(i) Two-person Zero-sum game
(ii) Principles of dominance
(iii) Pure strategy in game theory.

Q5- Explain the graphical method of solving 2xn and mx2 games.

Q6- For the following two persons zero sum game, find the optimal strategy for each player and the value of the game.

	Player B			
Player A	B1	B2	B3	B4
A1	0	1	3	5
A2	-5	2	4	5
A3	-2	-3	-4	-2

Ans-A-A_1, B-B_2, v=0

Q7- Define:

(i) Competitive game
(ii) Payoff matrix
(iii) Pure and mixed strategies

Q8- State the major limitations of the game theory. What are the assumptions made in the theory of game?

Q9- Which situation is called a game? What is the maximum criterion of optimality?

Q10- Discuss the following:

(i) Saddle point
(ii) Optimal strategies
(iii) Rectangular (or two person zero-sum) game

Q11: Define saddle point and solve the game whose pay- off matrix is

$$\begin{array}{cc} & \text{PlayerB} \\ \text{Player A} & \begin{pmatrix} -5 & 3 & 1 & 20 \\ 5 & 5 & 4 & 6 \\ -4 & -2 & 0 & -5 \end{pmatrix} \end{array}$$

Ans- A-A_1, B-B_1, v=0

Q12: Find the upper and lower bounds for the values of the following games:

(i) $$\begin{bmatrix} 3 & 7 & -1 & 3 \\ 4 & 8 & 0 & -6 \\ 6 & -9 & -2 & 4 \end{bmatrix}$$

$$
\text{(ii)} \quad \begin{bmatrix} -1 & 9 & 6 & 8 \\ -2 & 10 & 4 & 6 \\ 5 & 3 & 0 & 7 \\ 7 & -2 & 8 & 4 \end{bmatrix}
$$

Ans- (i) -1<v<0
(ii) 0<v<7

Q13: Use the notion of dominance to simplify the rectangular game with the following pay-off, matrix, and then solve it graphically.

$$
\text{(i)} \quad \begin{bmatrix} 19 & 15 & 17 & 16 \\ 0 & 20 & 15 & 8 \\ 19 & 15 & 17 & 16 \end{bmatrix} \qquad \text{(ii)} \quad \begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix}
$$

Ans: (i) A (15/16,1/16,0), B (0, 11/16, 0, 5/16), v=245/16
ii) A (0,0,1), B (4/5,1/5, 0) or (2/5, 3/5, 0), v=2

Q14: Find the range of values of p and q so that the entry (2,2) is a saddle point in the following games:

a)

2	q	4
p	6	11
7	3	4

b)

0	2	3
8	5	q
2	p	4

Ans: a) p ≥ 6, q≤6
b) p ≤5, q≥5

Q15- Solve the following games:

(i) B (ii) B

I II III

$$A\;\begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \end{array}\begin{bmatrix} -3 & -2 & 6 \\ 2 & 0 & 2 \\ 5 & -2 & -4 \end{bmatrix} \qquad\qquad A\;\begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \end{array}\begin{bmatrix} 1 & 3 & 1 \\ 0 & -4 & -3 \\ 1 & 5 & -1 \end{bmatrix}$$

iii)

$$\begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \text{V} \end{array}\begin{array}{c} \begin{array}{ccccc} \text{I} & \text{II} & \text{III} & \text{IV} & \text{V} \end{array} \\ \begin{bmatrix} 4 & 0 & 1 & 7 & -1 \\ 0 & -3 & -5 & -7 & 5 \\ 3 & 2 & 3 & 4 & 3 \\ -6 & 1 & -1 & 0 & 5 \\ 0 & 0 & 6 & 0 & 0 \end{bmatrix} \end{array}$$

iv)

$$\begin{array}{c} \\ \\ A \end{array}\;\begin{array}{c} \\ \\ \text{I} \\ \text{II} \\ \text{III} \end{array}\begin{array}{c} B \\ \begin{array}{cccc} \text{I} & \text{II} & \text{III} & \text{IV} \end{array} \\ \begin{bmatrix} -5 & 3 & 1 & 20 \\ 5 & 5 & 4 & 6 \\ -4 & -2 & 0 & -5 \end{bmatrix} \end{array}$$

Ans- (i) A-II, B-II, v=0   **ii) A-A_1, B-B_1 or B_2, v=1**
iii) A-III, B-II, v=2   **v) A-II, B-III, v=4**

Q16- Reduce the following games to (2×2) by graphical method and hence solve the games.

(i) B

$$A\begin{bmatrix} 1 & 4 & -2 & -3 \\ 2 & 1 & 4 & 5 \end{bmatrix}$$

ii) B

$$A\begin{bmatrix} 2 & -1 & 5 & -2 & 6 \\ -2 & 4 & -3 & 1 & 0 \end{bmatrix}$$

iii) B

$$A\begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix}$$

iv) B

$$A\begin{bmatrix} 2 & 1 & 0 & -2 \\ 1 & 0 & 3 & 2 \end{bmatrix}$$

Ans- (i) A (1/4, 3/4), B (3/4,1/4,0,0), v=7/4
(ii) A (3/7, 4/7), B (3/7,0,0,4/7,0), v= -2/7
(iii) A (3/11,8/11), B (0,9/11,2/11), v=49/11
iv) A (2/5,3/5), B (0, 4/5,0, 1/5), v=2/5

Q17- Solve the following games by linear programming i.e., by simplex method:

(i)

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 3 & -1 & -3 \\ -3 & 3 & -1 \\ -4 & -3 & 3 \end{bmatrix} \end{array}$$

(ii)

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 3 \\ -1 & 2 & -1 \end{bmatrix} \end{array}$$

Ans- (i) A (4/9, 11/45, 14/45), B (14/45, 11/45, 20/45), v=29/45

(ii) A (6/13, 3/13, 4/13), B (6/13,4/13,3/13), v= -1/13

Unit-IV: Integer Programming Problems

1) Explain the algorithm in **Branch and Bound method.**
2) What is Integer Programming Problem?
3) What is Mixed Integer Programming Problem?
4) Explain the Gomory's Cutting Plane Algorithm for all Integer Programming Problem.
5) What is difference between cutting plane method and Branch and Bound method?
6) What is difference between Integer Programming Problem and Linear Programming Problem?
7) Solve the following mixed integer programming problem (use branch and bound method).

   Max.:

   $$Z = 8X + 5Y$$

   Subject to: $9X + 5Y \leq 45$

   $X + Y \leq 6$

   $X \geq 0$, $Y \geq 0$ and X, Y are integers. [**Ans-(5,0) 40]**

8) Solve the following mixed integer programming problem. (Use branch and bound method)

Max.:

$$Z = 5X + 6Y$$

Subject to: $X + Y \leq 5$

$4X + 7Y \leq 28$

[Ans-(3,2) 27]

9) Find the solution of the following integer programming problem. (Use Cutting plane method)

Max. $Z = 2X_1 + 20X_2 - 10X_3$

Subject to the constraints:

$$2X_1 + 20X_2 + 4X_3 \leq 15$$

$$6X_1 + 20X_2 + 4X_3 = 20$$

$X_1 \geq 0$, $X_2 \geq 0$, $X_3 \geq 0$ and X_1, X_2, X_3 are integers

. **[Ans-(2,0,2) -16]**

10) Solve the following integer programming problem to obtain optimum integer solution (Use Cutting plane method).

Max.:

$$Z = 7X + 9Y$$

Subject to: $-X + 3Y \leq 6$

$7X + Y \leq 35$

$X \geq 0$, $Y \geq 0$ and X, Y are integers.

[Ans-(4,3) 55]

11) Solve the following integer programming problem by Cutting plane method.

Max. $Z = -3X_1 + X_2 + 3X_3$

Subject to the constraints:

$$-X_1 + 2X_2 + X_3 \leq 15$$

$$4X_2 - 3X_3 \leq 2$$

$$X_1 - 3X_2 + 2X_3 \leq 3$$

$X_1 \geq 0$, $X_2 \geq 0$, and X_1, X_3 are integers.

[Ans-(0,8/7,1) 29/7]

12) Solve the following integer programming problem by branch and bound technique.

Max.:

$$Z = X + Y$$

Subject to: $2Y \leq 7$

$X + Y \leq 7$

$2X \leq 1$

$X \geq 0$, $Y \geq 0$ and X, Y are integers.

[Ans-(4,3) 10]

13) Solve the following integer programming problem by branch and bound technique.

Max.:

$$Z = 2X + Y$$

Subject to: $X \leq 3/2$

$Y \leq 3/2$

$X \geq 0$, $Y \geq 0$ and X, Y are integers.

Ans- X=1, Y=1, max. z=3

14) Solve the following integer programming problem by Cutting plane method.

Max.: $Z = X + Y$

Subject to: $3X + 2Y \leq 5$

$Y \leq 2$

$X \geq 0$, $Y \geq 0$ and X, Y are integers.

Ans- X=0, Y=2, Mx. z= 2

15) Solve the following integer programming problem by branch and bound technique.

Max.: $Z = 2X + 3Y$

Subject to: $5X + 7Y \leq 35$

$4X + 9Y \leq 36$

$X \geq 0$, $Y \geq 0$ and X, Y are integers.

Ans- X=4, Y=2, Mx. z= 14